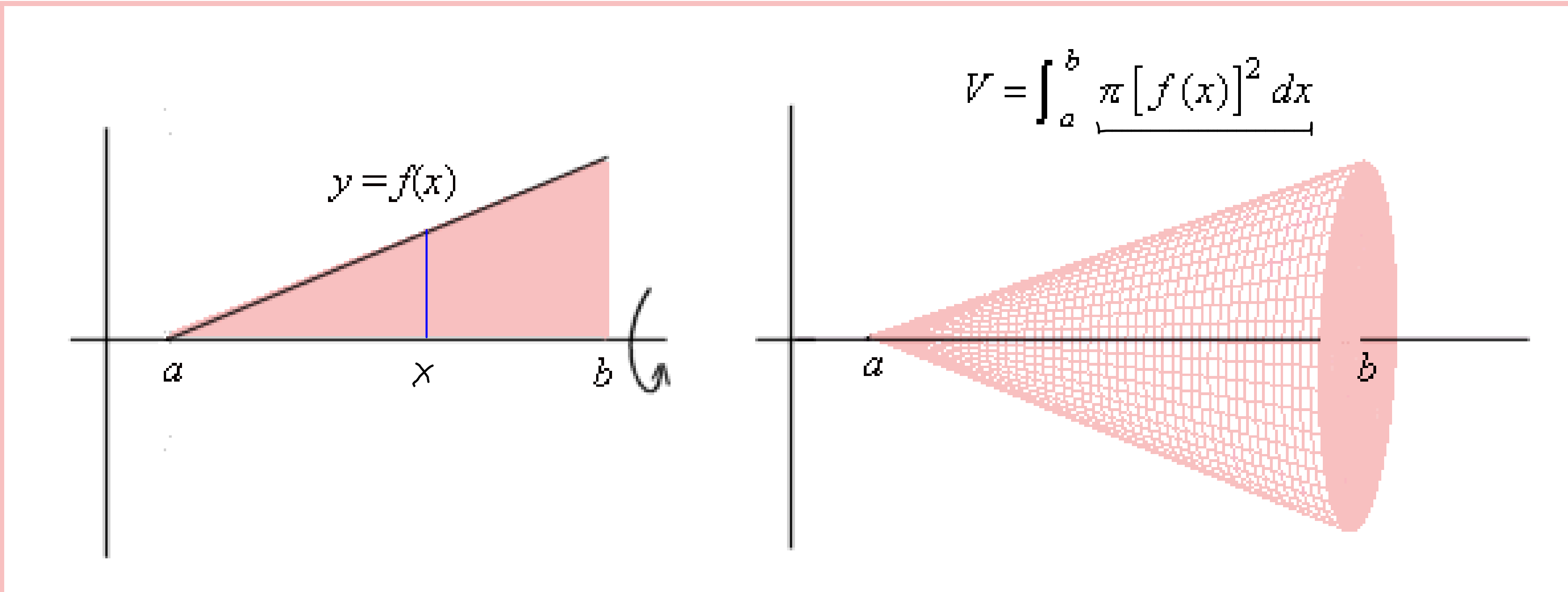
Today: *More* Section 6.2/6.3

- **Test 4 , Dec. 6 - 8**
- **Final Exam, Dec. 17 - 19**
- Dates are subject to change...
- **Practice Test 4** (counts as a quiz) and **Practice Final Exam** (counts as 2 quizzes) will be posted soon.
- **EMCFs** will be posted for the rest of the week, and **Homework** will be posted for *next* Monday.
- **Practice Problems** will be posted soon for Test 4.

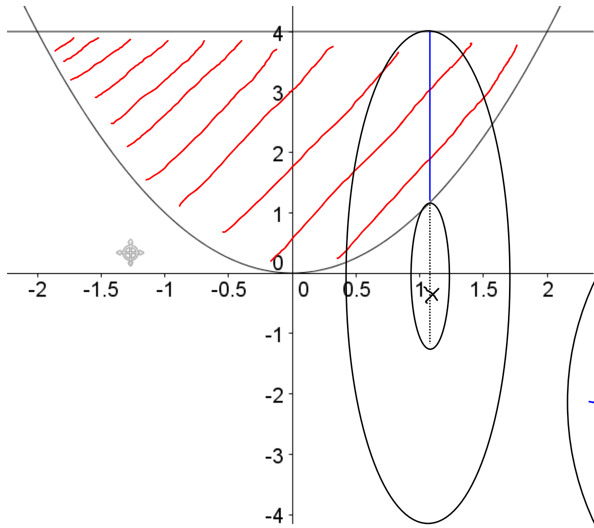
Recall: Revolving A Region About a Horizontal Axis

Note: The picture and formula shown is for the special case when the horizontal axis is the x-axis.

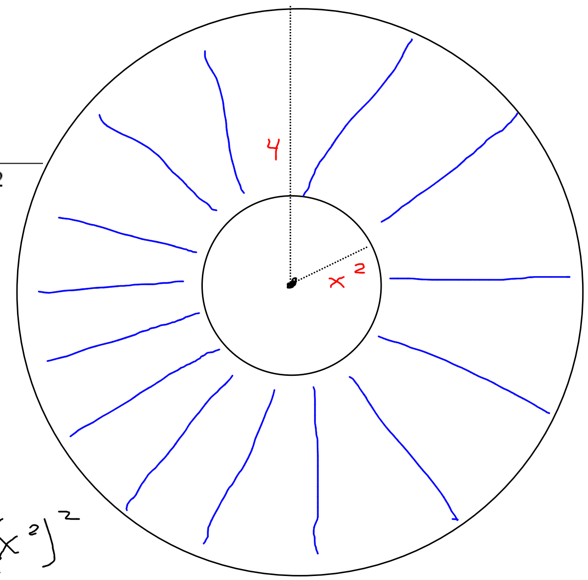


Idea for finding the volume that is generated: Fill up the region on the left with vertical line segments. Rotate each of these line segments around the horizontal axis. Determine the shape that is generated from rotating each line segment, and the infinitesimal volume of the shape. Sum up the infinitesimal volumes of these shapes with an integral.

Example: Revolve the region bounded by the line $y = 4$ and $y = x^2$ about the x-axis, and find the resulting volume.



washer



Thickness $= dx$

Face Area $= \pi \cdot 4^2 - \pi \cdot (x^2)^2$

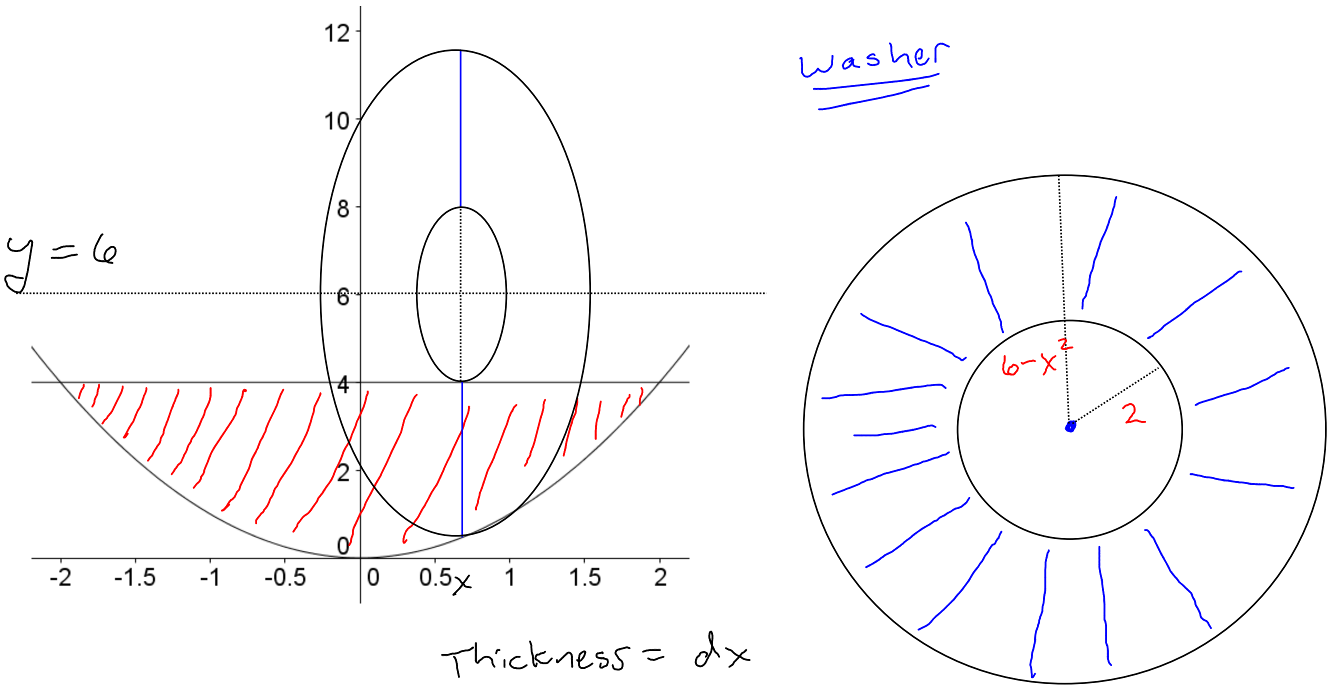
$= 16\pi - \pi x^4$

Inf. Volume (washer) $= (16\pi - \pi x^4)\, dx$

Full Volume $= \int_{-2}^{2} (16\pi - \pi x^4)\, dx$

$= \ldots$

Example: Revolve the region bounded by the line $y = 4$ and $y = x^2$ about the line $y = 6$, and find the resulting volume.



$$\text{Thickness} = dx$$

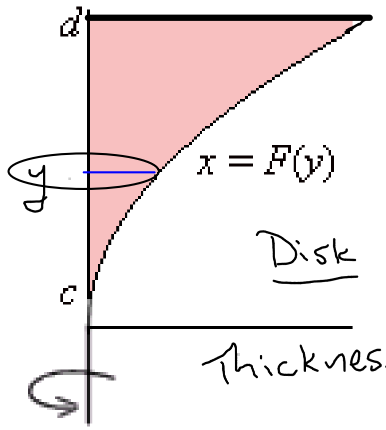
$$\text{Face Area} = \pi\left(6-x^2\right)^2 - \pi \cdot 2^2$$

$$\text{Inf. Volume (washer)} = \left(\pi\left(6-x^2\right)^2 - 4\pi\right)dx$$

$$\text{Full Volume} = \int_{-2}^{2}\left(\pi\left(6-x^2\right)^2 - 4\pi\right)dx = \cdots$$

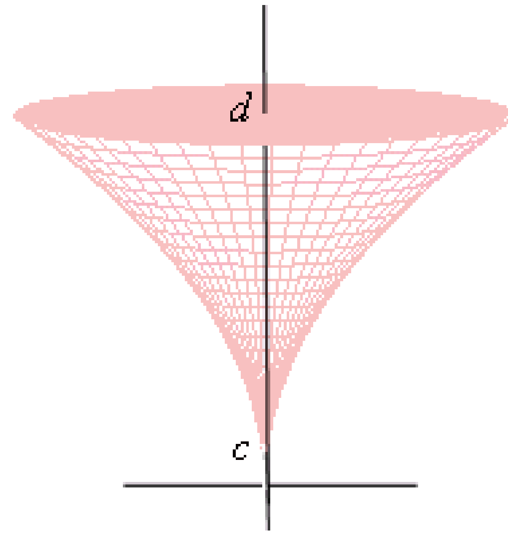
Revolving A Region About a Vertical Axis
(Part I)

Note: The picture and formula shown is for the case when the vertical axis is the y-axis.



Disk

Thickness = dy



Face Area = $\pi F(y)^2$

Inf. Volume = $\pi F(y)^2 dy$

$$\boxed{V = \int_c^d \pi \left[F(y)\right]^2 dy}$$

Idea for finding the volume that is generated: Fill up the region on the left with horizontal line segments. Rotate each of these line segments around the vertical axis. Determine the shape that is generated from rotating each line segment, and the infinitesimal volume of the shape. Sum up the infinitesimal volumes of these shapes with an integral.

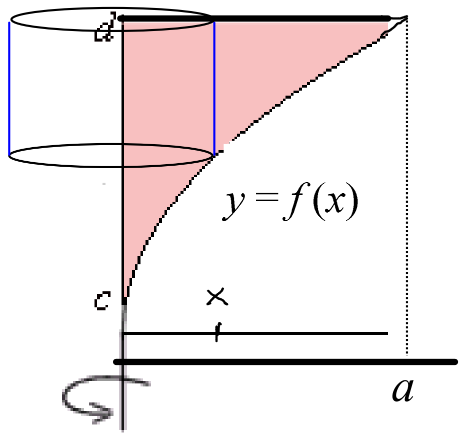
Revolving A Region About a Vertical Axis
(Part I)

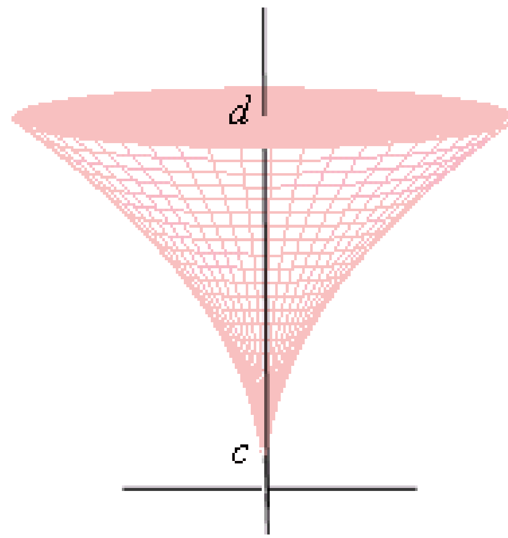
Note: The picture and formula shown is for the case when the vertical axis is the *y*-axis.

cylindrical shell

Thickness = dx

Area = $2\pi x\,(d - f(x))$





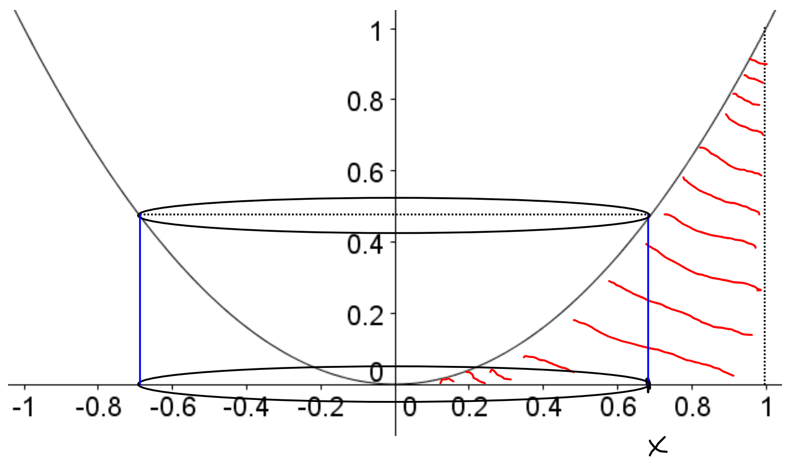
Inf volume (shell) $= 2\pi x\,(d - f(x))\,dx$

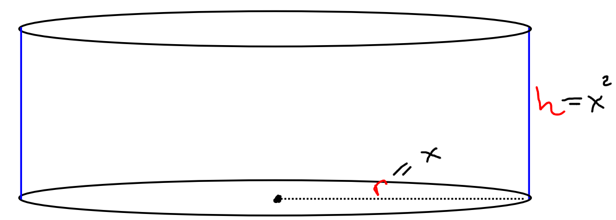
Full Volume $= \int_0^a 2\pi x\,(d - f(x))\,dx$

Idea for finding the volume that is generated: Fill up the region on the left with vertical line segments. Rotate each of these line segments around the vertical axis. Determine the shape that is generated from rotating each line segment, and the infinitesimal volume of the shape. Sum up the infinitesimal volumes of these shapes with an integral.

Example: Find the volume generated when the region bounded between the x-axis and the graph of $y = x^2$ on $[0,1]$ is rotated around the y-axis.

Cylindrical shell





Thickness = dx

Surface Area (shell) $= 2\pi r h$

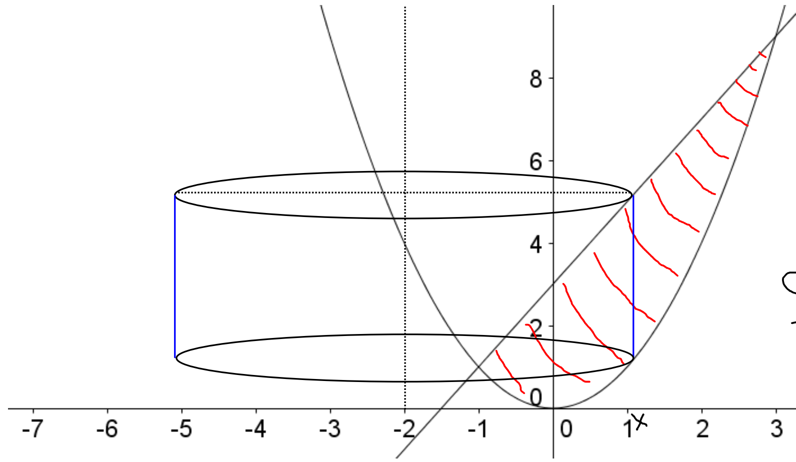
$= 2\pi x \cdot x^2$

$= 2\pi x^3$

Inf. Volume (shell) $= 2\pi x^3 \, dx$

Full Volume $= \int_0^1 2\pi x^3 \, dx = \ldots$

Example: A region in the *xy* plane is bounded by the curves $y = x^2$ and $y = 2x + 3$. Rotate this region about the line $x = -2$, and find the volume that is generated.



$x^2 = 2x + 3$

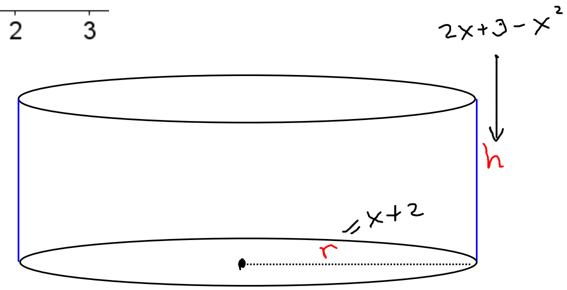
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, \; x = 3$

Cylindrical shell.

Thickness $= dx$



Surface Area (shell) $= 2\pi r h$

$= 2\pi(x+2)(2x+3-x^2)$

Inf. Volume (shell) $= 2\pi(x+2)(2x+3-x^2)\,dx$

Full Volume $= \displaystyle\int_{-1}^{3} 2\pi(x+2)(2x+3-x^2)\,dx = \cdots$

(multiply out)