Today: *More* Section 6.2/6.3

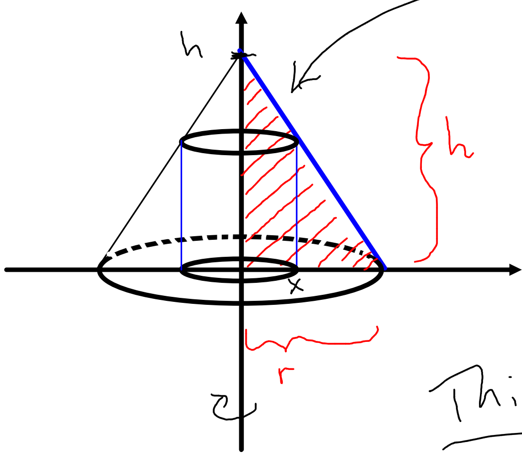
Dec. 3

- **Test 4 , Dec. 6 - 8**
- **Final Exam, Dec. 17 - 19**
- Dates are subject to change...
- **Practice Test 4** (counts as a quiz) and **Practice Final Exam** (counts as 2 quizzes) will be posted soon.
- **EMCFs** are posted for the rest of the week, and **Homework** is posted for Monday.
- **Practice Problems** will be posted soon for Test 4.
- **Friday's Quiz** will cover volumes of revolution.

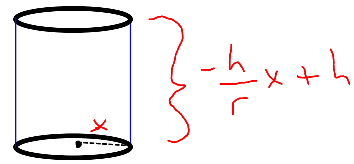
Last one

Example: Create a right circular cone with height h and base radius r as a volume of revolution. Then use your creation to derive the formula for the volume of a cone.

$$y = -\frac{h}{r}x + h$$



Cylindrical Shell



Thickness = dx

$$\text{Surface Area (shell)} = 2\pi x\left(-\frac{h}{r}x + h\right)$$

$$\text{Inf. Volume (shell)} = 2\pi x\left(-\frac{h}{r}x + h\right)dx$$

$$\text{Volume (cone)} = \int_0^r 2\pi x\left(-\frac{h}{r}x + h\right)dx$$

$$= \int_0^r \left(-\frac{2\pi h}{r}x^2 + 2\pi h x\right)dx$$

$$= \left(-\frac{2\pi h}{r}\cdot\frac{x^3}{3} + \pi h x^2\right)\Bigg|_0^r$$

$$= -\frac{2}{3}\pi h r^2 + \pi h r^2 - 0$$

$$= \frac{1}{3}\pi r^2 h$$

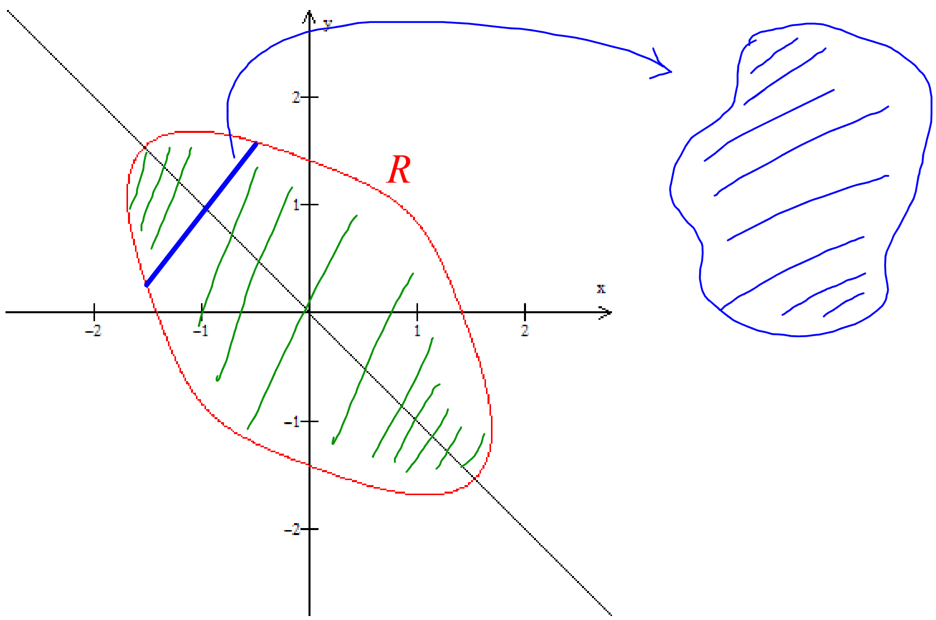
Popper P32

1. Give the volume of the solid generated when the region bounded between $y = x^3$ and the x-axis on the interval $[0,1]$ is rotated around the x-axis.

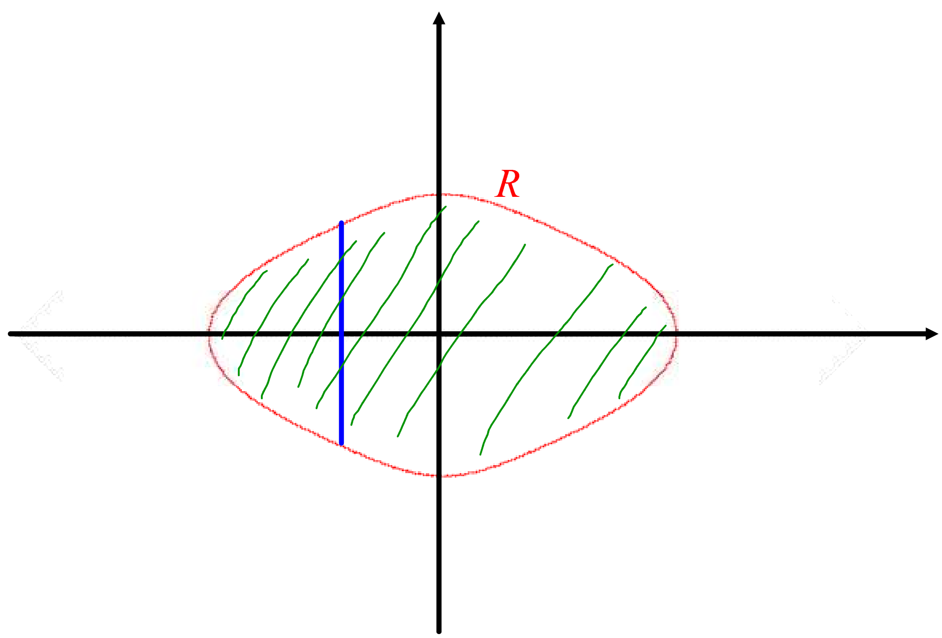
2. Give the volume of the solid generated when the region bounded between $y = x^3$ and the x-axis on the interval $[0,1]$ is rotated around the y-axis.

Volumes by Slicing: Main Setting

A solid S intersects the xy-plane in the region R shown below. Every cross section of the solid S taken perpendicular to a given line can be described. **Find the volume of S.**



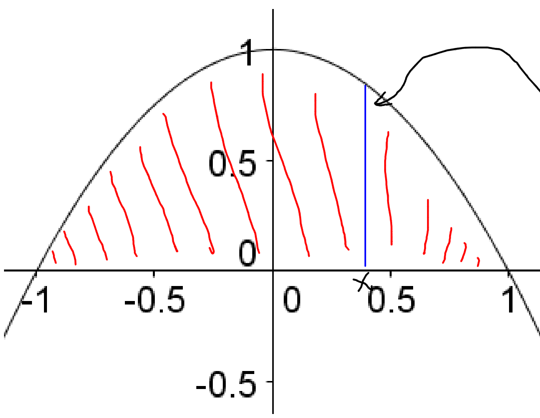
Typically, the solid object is aligned so that the cross sections perpendicular to one of the axes is known.



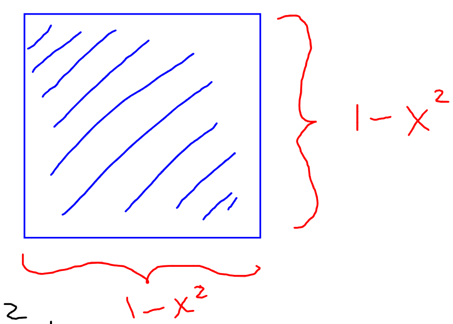
Example: A solid object is aligned so that its base is the region in the *xy*-plane bounded between the *x*-axis and the curve $y = 1 - x^2$. Cross sections taken perpendicular to the *x*-axis are squares. Find the volume of the region.

P32

3. Give the volume.



Pull the cross section out, and lay it flat.



Thickness $= dx$

Area $= (1-x^2)^2$

Inf. Volume (cross section) $= (1-x^2)^2\,dx$

Full Volume $= \displaystyle\int_{-1}^{1} (1-x^2)^2\,dx$

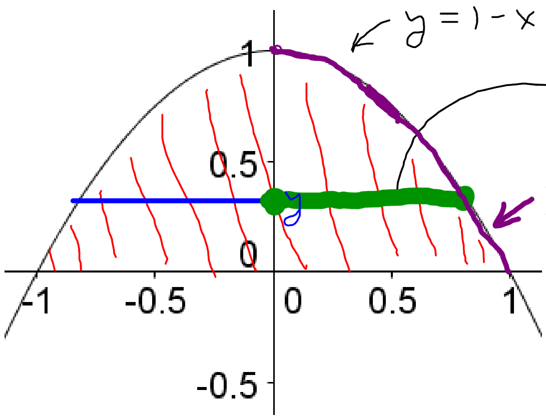
Example: A solid object is aligned so that its base is the region in the *xy*-plane bounded between the *x*-axis and the curve $y = 1 - x^2$. Cross sections taken perpendicular to the *y*-axis are semi-circles with their diagonals lying in the *xy*-plane. Find the volume of the region.

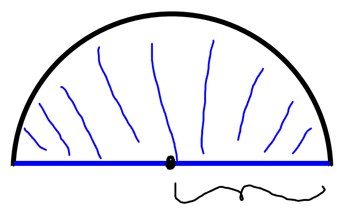
P32

4. Give the volume.

$y = 1 - x^2 \Rightarrow x^2 = 1 - y$

$x = \pm\sqrt{1-y}$





$r = \sqrt{1-y}$ ← in terms of y

Thickness $= dy$

Area $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(1-y)$

Inf Volume (cross section) $= \frac{1}{2}\pi(1-y)\,dy$

Full Volume $= \int_0^1 \frac{1}{2}\pi(1-y)\,dy =$