Homework Help Session

4.7: 1, 2, 3, 4, 5, 6, 7, 10, 11, 16, 17, 18, 19, 20, 22, 23, 25, 26, 28, 29

4.8: 1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 29, 30, 31, 34

(3) 4.7 $f(x) = \dfrac{x}{3x-1}$ ← rational function

Horiz. Asympts.:

$$\lim_{x\to\infty} \frac{x}{3x-1} = \frac{1}{3}$$

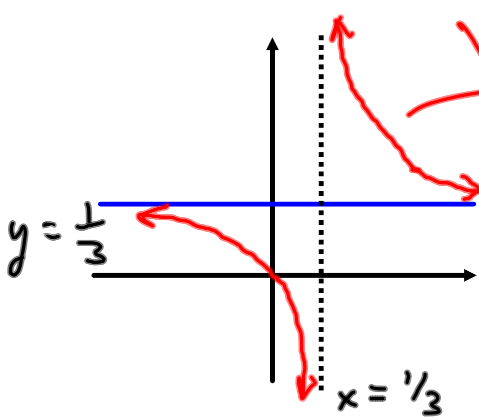
$$\lim_{x\to-\infty} \frac{x}{3x-1} = \frac{1}{3}$$

Vert. Asympts.: set $3x-1=0$

$$x = \frac{1}{3}$$

Answer: Horiz. Asympt.

$y = 1/3$



Vert Asympt.

$x = 1/3$.

(17) $^{4.7}$ $f(x) = \sqrt{x+4} - \sqrt{x}$

Only defined for $x \geq 0$.

No vert. asympt.

Horiz:

$$\lim_{x\to\infty} \left(\sqrt{x+4} - \sqrt{x}\right)$$

$$= \lim_{x\to\infty} \overset{a\ -\ b}{\left(\sqrt{x+4} - \sqrt{x}\right)} \frac{\overset{a\ +\ b}{\left(\sqrt{x+4} + \sqrt{x}\right)}}{\left(\sqrt{x+4} + \sqrt{x}\right)}$$

$$= \lim_{x\to\infty} \frac{x+4-x}{\sqrt{x+4} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{4}{\sqrt{x+4} + \sqrt{x}} = 0$$

$y = 0$ is a horiz asympt.

(19) 4.7 $f(x) = \dfrac{\sin(x)}{\sin(x)-1}$

Horizontal: $\lim_{x\to\infty} \dfrac{\sin(x)}{\sin(x)-1} =$ No Limit

$\lim_{x\to-\infty} \dfrac{\sin(x)}{\sin(x)-1} =$ No Limit

Vert. Asympt: Find vals. of x where denom. $= 0$
num. $\neq 0$.

ie. $\underbrace{\sin(x)-1=0}_{\sin(x)=1}$ and $\underbrace{\sin(x)\neq 0}_{\text{no prob.}}$

All vert asympt.

$x = \pi/2,\ \frac{\pi}{2}+2\pi,\ \pi/2+4\pi, \ldots$
$\pi/2-2\pi,\ \pi/2-4\pi, \ldots$

(22) 4.7

$$f(x) = 3 + x^{2/5}$$

$$f'(x) = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}}$$

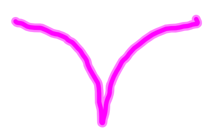
$x = 0$ is a c.n.

f' dne. (f' becomes infinite near 0.)

$f'(x)$	$-\ -$	$-$	dne	$+\ +$	$+\ +$
x		-1	0	1	

$f'(-1) = $ neg.

$f'(1) = $ pos.

 vert. cusp.

(23) 4.7

$$f(x) = (2-x)^{4/5} = (x-2)^{4/5}$$

$$f'(x) = \frac{4}{5}(x-2)^{-1/5} = \frac{4}{5(x-2)^{1/5}}$$

c.n. at $x=2$ b/c $\underbrace{f'(2) \text{ d.n.e.}}_{}$

f' becomes infinite near $x=2$.

$f'(x)$: $-$ $-$ $-$ | d.n.e. | $+$ $+$ $+$

x: 0, 2, 3

$f'(0) = \text{neg.}$ $\qquad$ $f'(3) = \text{pos.}$

vert. cusp.

4.8 Graph.

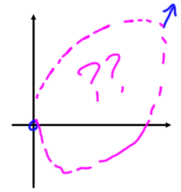
6. $f(x) = x^4 - 8x^2, \quad x > 0$

1. Domain: $x > 0$.

2. Asympt. and large x.

No vert asympt.

$\lim_{x \to \infty} (x^4 - 8x^2) = \infty$



$\lim_{x \to 0^+} (x^4 - 8x^2) = 0$

3. $f'(x) = 4x^3 - 16x, \quad x > 0$

$f'(x) = 0 \Leftrightarrow 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = \cancel{0}, \quad x = \cancel{\pm}2$

I still see it.

$f'(x)$	0	– – –		0	+ + + + +
x	0		1	2	3

$f'(1) < 0 \qquad f'(3) > 0$

Dec: $(0, 2]$ Inc: $[2, \infty)$

loc. min. at 2.

4. $f''(x) = 12x^2 - 16, \quad x > 0$

Set $f''(x) = 0, \quad 12x^2 - 16 = 0$

$x^2 = \frac{4}{3}$

$x = \cancel{\pm} \frac{2}{\sqrt{3}}$

$f''(x)$	– – – – – –		0	+ + + + +
x	0	1	$\frac{2}{\sqrt{3}}$	3

$f''(1) < 0 \qquad f''(3) > 0$

C.D. $\left(0, \frac{2}{\sqrt{3}}\right]$ C.U. $\left[\frac{2}{\sqrt{3}}, \infty\right)$

Inflection: $x = \frac{2}{\sqrt{3}}$

5. Plot: $x = \frac{2}{\sqrt{3}}$, 2

$(2, -16)$

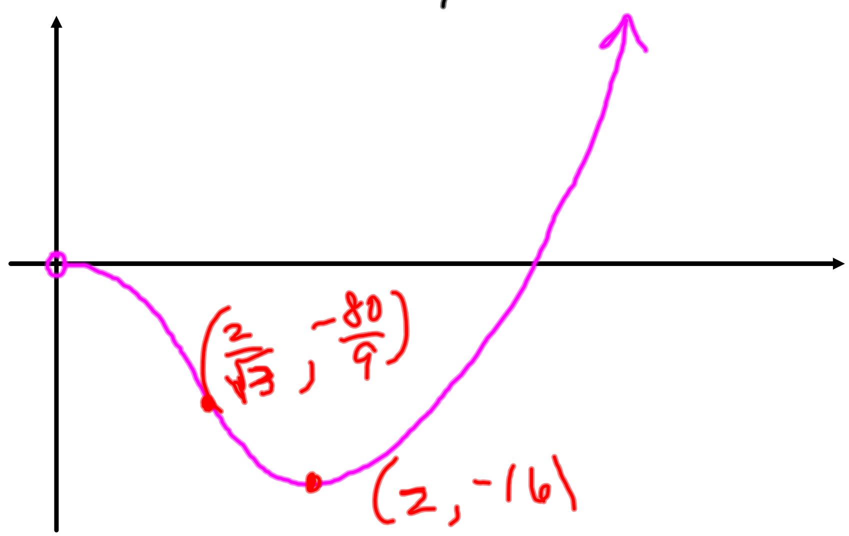
$\left(\frac{2}{\sqrt{3}}, \frac{-80}{9}\right)$

$f(x) = x^4 - 8x^2$

$f(2) = -16$

$f\left(\frac{2}{\sqrt{3}}\right) = \frac{16}{9} - 8 \cdot \frac{4}{3}$

$= \frac{16 - 96}{9} = \frac{-80}{9}$



22. $f(x) = \dfrac{2x^2}{x+1}$ ← rational function.

1. Domain: All x except -1.

2. Asympt. + behavior for large x.

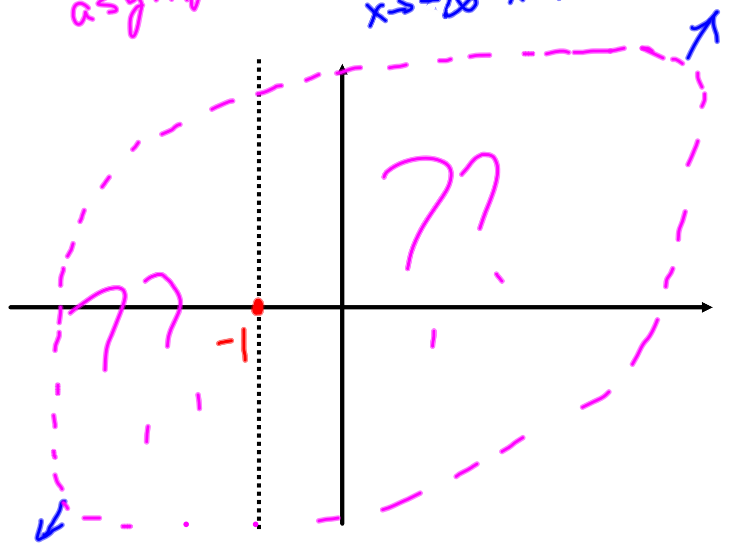
vert: $x+1=0$, $\boxed{x=-1}$

Vert asympt. $x=-1$.

horiz: $\lim\limits_{x\to\infty} \dfrac{2x^2}{x+1} = \infty$

$\lim\limits_{x\to-\infty} \dfrac{2x^2}{x+1} = -\infty$

No horiz. asympt.



3. $f(x) = \frac{2x^2}{x+1}$

$$f'(x) = \frac{(x+1)\cdot 4x - 2x^2}{(x+1)^2}$$

$$= \frac{4x^2 + 4x - 2x^2}{(x+1)^2}$$

$$f'(x) = \frac{2x^2 + 4x}{(x+1)^2} \leftarrow \text{pos.}$$

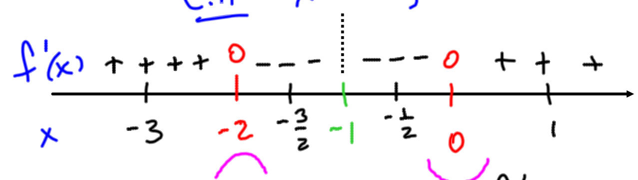
Note: $f'(-1)$ d.n.e, but -1 is not in the domain. $\therefore -1$ is not a c.n.

Check $f'(x) = 0$.

$2x^2 + 4x = 0$

$2x(x+2) = 0$

c.n. $x = 0, -2$.



$f'(-3) > 0$ $f'(-\frac{3}{2}) < 0$ $f'(1) > 0$

$f'(-\frac{1}{2}) < 0$

Inc: $(-\infty, -2]$, $[0, \infty)$

Dec: $[-2, -1)$, $(-1, 0]$

$x = -2$ loc. max.

$x = 0$ loc. min.

4. $f'(x) = \dfrac{2x^2+4x}{(x+1)^2}$

$$f''(x) = \frac{(x+1)^{\cancel{2}\,1}\cdot(4x+4) - (2x^2+4x)\,2\,\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{4x^2}+\cancel{8x}+4-\cancel{4x^2}-\cancel{8x}}{(x+1)^3}$$

$$= \frac{4}{(x+1)^3}$$

← never 0. exists except at $x=-1$, which is not in domain.

$f''(x)$	$- - -$	$- - -$		$+ + + +$	
x		-2	-1		0

$f''(-2) < 0$ $\qquad$ $f''(0) > 0$

C.D. $(-\infty, -1)$, C.U. $(-1, \infty)$

5. Graph. $f(x) = \frac{2x^2}{x+1}$

Plot: $x = -2$, $x = 0$

$f(-2) = -8$, $f(0) = 0$

$(-2, -8)$, $(0, 0)$