1. Use differentials to estimate $\sqrt{102}$.

Note: $\sqrt{100} = 10$

$a = 100$

$h = 102 - 100 = 2$

$f(x) = \sqrt{x}$

$df = f'(a)h$

$f'(x) = \frac{1}{2\sqrt{x}}$

$\Updownarrow$

$f(102) - f(100) \approx f'(a)h$

$\sqrt{102} - 10 \approx \frac{1}{20} \cdot 2$

$\sqrt{102} \approx 10 + \frac{1}{10} = 10.1$ #

2. Use differentials to estimate the maximum amount the radius of a circle can change in order to keep the area of the circle within .01 of 4π.

$A(r) = \pi r^2 \qquad A'(r) = 2\pi r$

Note: Area $= 4\pi = \pi r^2$ $\Big\}$ $r = 2$.

$dA = A'(2)h$

$.01 = 4\pi h \Rightarrow h = \frac{.01}{4\pi}$ #

3. Verify the Mean Value Theorem for the function

$f(x) = x^3 - 4x^2 - x$ on the interval $[0, 2]$.

$f'(x) = 3x^2 - 8x - 1$

Find c so that $0 < c < 2$

and $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$

$3c^2 - 8c - 1 = \dfrac{8 - 16 - 2 - 0}{2}$

$3c^2 - 8c - 1 = -5$

$3c^2 - 8c + 4 = 0$ $\quad c = \dfrac{8 \pm \sqrt{64 - 48}}{6}$

$c = \dfrac{8 \pm 4}{6}$ $\quad \cancel{c = 2}$ or $c = \dfrac{2}{3}$ ✓

4. Find and classify the critical numbers of

$f(x) = 3x\sqrt{4-x}$.

Domain: Need $4 - x \geq 0$

$x \leq 4$.

$$f'(x) = 3\sqrt{4-x} + 3x \frac{1}{2\sqrt{4-x}}(-1)$$

$$= \frac{6(4-x) - 3x}{2\sqrt{4-x}} = \frac{24 - 9x}{2\sqrt{4-x}}$$

C.N.: $\boxed{x=4}$. AND $24 - 9x = 0 \Leftrightarrow \boxed{x = \frac{8}{3}}$

$f'(x)$	+ + + +	0	− − − −	DNE
x	0	$\frac{8}{3}$	3	4

loc. max at $x = 8/3$

loc. min at $x = 4$.

5. Find the absolute minimum value of the function $f(x) = 3x^4 + 16x^3 + 24x^2 - 1$ on the interval $[-1,1]$.

1. $f(-1) = 3 - 16 + 24 - 1$

$= 10$

$f(1) = 3 + 16 + 24 - 1 = 42$

2. Get C.N. in $[-1, 1]$.

$f'(x) = 12x^3 + 48x^2 + 48x$

$= 12x(x^2 + 4x + 4)$

$= 12x(x+2)^2$

$f'(x) = 0 \Leftrightarrow \boxed{x = 0}$ or $\cancel{x = -2}$

$f(0) = -1.$

$\therefore$ Comparing $f(-1)$, $f(1)$ and $f(0) \Rightarrow f$ has its abs. min on $[-1, 1]$ at $x = 0$.

6. Find and classify the critical numbers of the function $f(x) = 3x^4 + 16x^3 + 24x^2 - 1$, and list the intervals of increase and decrease.

$$f'(x) = 12x(x+2)^2$$

$$f'(x) = 0 \iff x = 0 \quad \text{or} \quad x = -2.$$

C.N.: $x = 0$, $x = -2$.

$f'(x)$	$- - - - -$	0	$- - - - -$	0	$+ + +$	
x	-3	-2	-1	0	1	

$x = -2$ is neither a loc. max nor a loc. min

$x = 0$ is a local min.

f is decreasing on $(-\infty, 0]$.

f is increasing on $[0, \infty)$.

7. Give the intervals of concavity of the function $f(x) = 3x^4 + 16x^3 + 24x^2 - 1$, and list any values where inflection occurs.

$$f'(x) = 12x(x+2)^2$$

$$f''(x) = 12(x+2)^2 + 12x \cdot 2(x+2)$$

$$= 12(x+2)\left[(x+2) + 2x\right]$$

$$= 12(x+2)(3x+2)$$

Candidates for infl. $x = -2$, $x = -\frac{2}{3}$

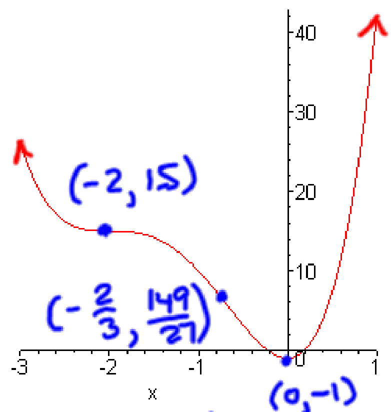
$f''(x)$	+ + +		0	−		− −	0	+ + +
x		-3	-2		-1		$-\frac{2}{3}$	0

Inflection at $x=-2$ and $x = -\frac{2}{3}$.

C.U.: $(-\infty, -2]$ or $[-\frac{2}{3}, \infty)$.

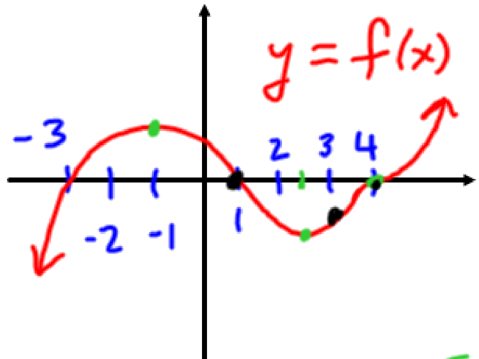
C.D.: $[-2, -\frac{2}{3}]$

8. Graph the function $f(x) = 3x^4 + 16x^3 + 24x^2 - 1$.



Use the information from #6 and #7.

9.



The graph of f is shown. Find and classify the critical numbers for f, list the intervals of increase and decrease, list the intervals of concavity, and give any values where inflection occurs.

C.N.: $x = -1, \frac{5}{2}, 4$

Incr: $(-\infty, -1]$ or $[\frac{5}{2}, \infty)$

Decr: $[-1, \frac{5}{2}]$

$x = -1$ is a loc. max
$x = 5/2$ is a loc. min.

$x = 4$ is neither

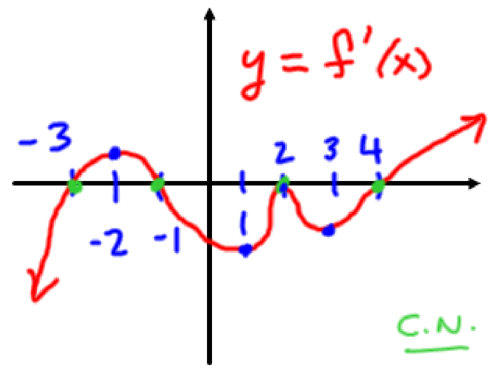
Inflection at $x = 1, 3, 4$

CD: $(-\infty, 1]$ or $[3, 4]$

CU: $[1, 3]$ or $[4, \infty)$.

Oct 25-7:06 AM

10.



The graph of f' is shown. Find and classify the critical numbers for f, list the intervals of increase and decrease, list the intervals of concavity, and give any values where inflection occurs.

C.N. $x = -3, -1, 2, 4$

$f'(x)$	- - 0 + + 0 - - - 0 - - 0 + + +
x	-3 -1 2 4
f	∪ ∩ ∽ ∪

loc min at $x = -3, 4$

loc. max at $x = -1$

Neither at $x = 2$.

Incr: $[-3,-1]$ or $[4, \infty)$

Decr: $(-\infty, -3]$ or $[-1, 4]$

$f'' > 0$?

$(f')'$

$f''(x)$	+ + + - - - + + - - - + +
x	-2 1 2 3

CU: $(-\infty, -2]$ or $[1, 2]$ or $[3, \infty)$

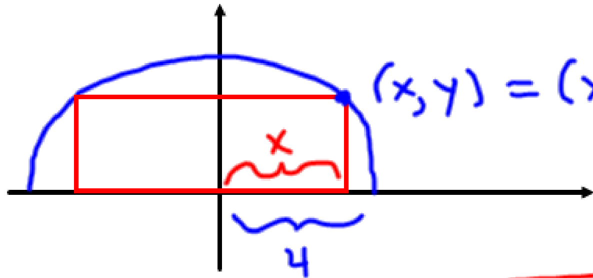
C.D.: $[-2, 1]$ or $[2, 3]$

Inflection at $x = -2, 1, 2, 3$.

12. Give the dimensions of the rectangle of largest area that can be inscribed in a semi-circle of radius 4.

$x^2+y^2=16$

$y=\sqrt{16-x^2}$



$\text{Area} = 2x\cdot y = 2x\sqrt{16-x^2}, \quad 0\le x\le 4$

$= A(x)$

1. $A(0)=0, \quad A(4)=0$

2. $A'(x) = 2\sqrt{16-x^2} + 2x\cdot\frac{1}{2\sqrt{16-x^2}}(-2x)$

$$= \frac{4(16-x^2)-4x^2}{2\sqrt{16-x^2}}$$

$$= \frac{64-8x^2}{2\sqrt{16-x^2}}$$

$A'(x)=0 \iff 64=8x^2 \quad x^2=8$

$x=2\sqrt{2}$ (is btwn 0 and 4)

$A(2\sqrt{2}) = 2\cdot 2\sqrt{2}\sqrt{16-8}$

$= 4\sqrt{2}\sqrt{8} = 16.$ ← ABS. MAX