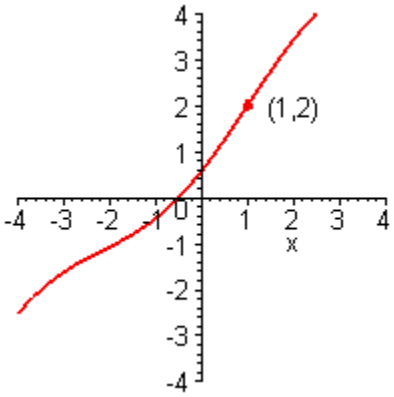
Math 1432

Final Exam Review

1. Graph the following and give the domain and range for $f(x)$.
   a. $f(x) = e^x$
   b. $f(x) = \ln(-x)$
   c. $f(x) = \arcsin(x)$
   d. $f(x) = \cosh(x)$
2. For each of the parts for problem 1, find $f'(x)$ and $\int f(x)dx$.
3. Give the equation of the tangent line to the given graph at the point where x = 0
   a. $f(x) = \ln(6x+1) + e^{2x}$
   b. $f(x) = \ln(2x+1) - 3e^{-4x}$
   c. $f(x) = \sqrt{9-x^2}$
4. Find the inverse of the following, if possible:
   a. $f(x) = \dfrac{2}{3-x}$
   b. $f(x) = \dfrac{x+1}{x+2}$
5. Find the derivative of the inverse for the following:
   a. $f(x) = x^3 + 1, \quad f(2) = 9, \quad \left(f^{-1}\right)'(9) =$
   b. $f(-3) = 1,\ f(1) = 2,\ f'(-3) = 3,\ f'(1) = -2, \quad \left(f^{-1}\right)'(1) =$
   c. $f(x)$ passes through the points (3 , -2) and (-2 , 1). The slope of the tangent line to the graph of $f(x)$ at $\boldsymbol{x}$ = 3 is -1/4. Evaluate the derivative of the inverse of f at -2.

6. The graph of $f(x)$ is shown below and $\frac{d}{dx} f^{-1}(2) = \frac{5}{6}$.



   a. $f'(1) =$

   b. $f(-2) = -1$ and $f'(-2) = 1/2$. Give $\frac{d}{dx} f^{-1}(-1)$.

7. Simplify $\tan(\arcsin(3x))$ for $0 < x < 1/3$.

8. Find the largest interval containing $x = -2$ on which the function $f(x) = x^3 - 9x^2 + 24x$ is invertible.

9. Find the equation of the tangent and the normal lines to the parametric curves at the given points:

   a. $x(t) = -2\cos 2t,\ y(t) = 4 + 2t,\ (-2,4)$

   b. $x(t) = 3\cos(3t) + 2t,\ y(t) = 1 + 5t,\ (3,1)$

10. Give an equation relating x and y for the curve given parametrically by

   a. $x(t) = -1 + 3\cos t \quad y(t) = 1 + 2\sin t$

   b. $x(t) = -1 + 3\cosh t \quad y(t) = 1 + 2\sinh t$

   c. $x(t) = -1 + 4e^t \quad y(t) = 2 + 3e^{-t}$

11. Integrate:

   a. $\int (\cosh(3x) + \sinh(2x))dx$

   b. $\int \frac{\cosh(x)}{1 + \sinh(x)} dx$

   c. $\int e^x \sin(e^x) dx$

   d. $\int_0^{\pi} \sin(x)\cos^2(x) dx$

   e. $\int_0^2 \sqrt{4 - x^2}\, dx$

   f. $\int 4^{3x} dx$

g. $\int \left(2^{7x} - \sinh(5x)\right) dx$

h. $\int \frac{\sin(3x)}{16 + \cos^2(3x)} dx$

i. $\int \frac{6x}{4 + x^4} dx$

j. $\int \tan(3x) dx$

k. $\int \frac{\arctan(x)}{1 + x^2} dx$

l. $\int \frac{\arctan(3x)}{1 + 9x^2} dx$

m. $\int \frac{1}{\sqrt{4 + x^2}} dx$

n. $\int \sqrt{9 - x^2}\, dx$

o. $\int 3\ln(4x) dx$

p. $\int x^2 e^x dx$

q. $\int \frac{5x + 14}{(x+1)(x^2 - 4)} dx$

r. $\int \frac{x^2 + 5x + 2}{(x+1)(x^2 + 1)} dx$

s. $\int \frac{2x^2}{\sqrt{9 - x^2}} dx$

t. $\int 2\arctan(10x) dx$

u. $\int 3x\cos(2x) dx$

12. A culture of bacteria is growing in such a way that the number of bacteria is changing at a rate proportional to the number of bacteria. If there are initially 10,000 bacteria, and 12,000 bacteria are present six hours later, what is the doubling time for the culture? (give your answer in terms of ln)

13. Give the solution to $\frac{dy}{dx} = -3y$, $y(0) = -2$.

14. Identify the geometric shape given by the parameterization

$x(t) = -2 + 3\cos(t), y(t) = 1 + 3\sin(t)$

15. Give a parameterization for the line segment from the point (1,6) to the point (-3,1).

16. Give a parameterization for the curve given in polar coordinates by $r = 1 + \sin(\theta)$.

17. Give the formula for the arc length of a curve parameterized by $x(t) = \cos(t)$, $y(t) = t^2$ for $0 \le t \le 1$.

18. Write the line $y = x$ in polar coordinates.

19. Use long division to rewrite $\dfrac{x^4}{x^3 + x^2 + 1}$

20. Give the partial fraction decomposition for $\dfrac{2x+1}{(x-1)^2(x^2+1)}$

21. Give the greatest lower bound of the set $\left\{ x \middle| x^2 + 3x - 10 < 0 \right\}$.

22. Does the sequence converge or diverge?

    a. $\left\{ \dfrac{2n^2 + 1}{3n^3 + 4n^2 + 6} \right\}$

    b. $\left\{ \dfrac{1}{n\ln(n)} \right\}$

23. Give the limit of the sequence

    a. $\{n\sin(1/n)\}$

    b. $\left\{ \left(\dfrac{n-1}{n} \right)^n \right\}$

24. Give the exact value of

    a. $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{4^n}$

    b. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(n+1)}$

25. Determine if the following series (A) converge absolutely, (B) converge conditionally or (C) diverge.

a. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+3}$

b. $\sum_{n=1}^{\infty} \frac{\cos \pi n}{n^2}$

c. $\sum_{n=0}^{\infty} \frac{4n(-1)^n}{3n^2+2n+1}$

d. $\sum_{n=0}^{\infty} \frac{3(-1)^n}{\sqrt{3n^2+2n+1}}$

e. $\sum_{n=0}^{\infty} \frac{3n(-1)^n}{\sqrt{3n^2+2n+1}}$

f. $\sum_{n=0}^{\infty} \left(4(-1)^n \left(\frac{n}{n+3} \right)^n \right)$

g. $\sum_{n=0}^{\infty} \left(\frac{2(-1)^n \arctan n}{3+n^2+n^3} \right)$

h. $\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3^n}{4^n+3n} \right)$

i. $\sum_{n=0}^{\infty} \left(\frac{(-1)^n 3}{(n+2)\ln(n+2)} \right)$

j. $\sum_{n=2}^{\infty} \frac{(-1)^n n!}{(n+1)!}$

k. $\sum_{n=2}^{\infty} \frac{(-1)^n}{3n+2}$

l. $\sum_{n=0}^{\infty} \frac{(-1)^n 10n^2}{3^n}$

m. $\sum_{n=2}^{\infty} \frac{(-1)^n 3^n}{n!}$

n. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+3n+2}$

o. $\sum_{n=2}^{\infty} \frac{\cos(\pi n) n^n}{n!}$

p. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

26. State the indeterminate form and compute the following limits :

a. $\lim_{n\to\infty} \frac{\ln(n+4)}{n+2}$

b. $\lim_{n\to\infty} (3n)^{\frac{2}{n}}$

c. $\lim_{n\to\infty} \left(1+\frac{3}{n}\right)^{2n}$

d. $\lim_{n\to 0} \frac{x-\sin(2x)}{x+\sin(2x)}$

e. $\lim_{x->0} \frac{e^{x^2}-1}{2x^2}$

f. $\lim_{x->0+} \left(\frac{1}{x}\right)^x$

g. $\lim_{x->0} \frac{3e^{x/3}-(3+x)}{x^2}$

h. $\lim_{x->\infty} \frac{x^2}{\ln x}$

i. $\lim_{x->0} \frac{1+x-e^x}{x(e^x-1)}$

j. $\lim_{x->0} \frac{\arctan(4x)}{x}$

27. Give the derivative of each power series below:

a. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{n^2+2}$

b. $\sum_{n=0}^{\infty} \frac{x^n}{2n+1}$

28. For each of the problems in number 27, give the antiderivate F of the power series so that F(0)=0.

29. Suppose $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n-1}}{(2n)!}$. Give the 13th derivative of f at $x = 0$.

30. Give the 5th degree Taylor polynomial for e^x centered at 0.

31. Give the 6th degree Taylor polynomial for $\cos(x)$ centered at 0.

32. Give the Taylor series expansion for $f(x) = e^{-x}$ centered at 0.

33. Give a power series expansion for $f(x) = \ln(x)$ centered at 1.

34. Give a power series expansion for $f(x) = \sin(3x)$ centered at 0.

35. Give a power series expansion for $f(x) = \frac{1}{(1+x)^2}$ centered at 0.

36. Find the smallest value of n so that the nth degree Taylor Polynomial for $f(x) = \ln(1+x)$ centered at x = 0 approximates $\ln(2)$ with an error of no more than 0.001 (also be able to do this with some of the other Taylor Polynomials)

37. $f(1) = -1,\ f'(1) = 2,\ f''(1) = -1$. Give the 2nd degree Taylor polynomial for f centered at 1.

38. Rewrite $f(x) = x^3 + 2x^2 - x + 1$ in powers of $(x+1)$.

39. Evaluate each improper integral:

a. $\int_0^{27} x^{-2/3}$

b. $\int_0^4 \frac{1}{\sqrt{4-x}}$

c. $\int_{-2}^{0} \frac{1}{x+1} dx$

40. Find the formula for the area of $r = 1 + 2\sin\theta$

    a. Inside inner loop

    b. Inside outer loop but outside inner loop

    c. Inside outer loop and below x-axis

41. Find the radius of convergence and interval of convergence for the following Power series:

    a. $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)3^{n+1}}$

    b. $\sum_{n=0}^{\infty} \frac{1}{3^n}(x-1)^n$

    c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{4^n}$

    d. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n n!}{n^n}$

    e. $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$

42. Give a power series representation for $\arctan(2x)$ and give the radius of convergence.

43. Give a value of n so that the Taylor polynomial of degree n for $f(x) = \sin(x)$ centered at 0 can be used to approximate $f(x)$ within 10^{-4} on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

44. Use logarithmic differentiation to find the derivative of:

    a. $y = (3x-1)^{\sin(x)}$

    b. $y = (x+1)^{\ln(x)}$

    c. $y = (x^2+2)^{(\frac{1}{\ln x})}$

45. Determine the convergence or divergence for each series with the given general term:

Series	Converge or Diverge?	Test used
$\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$		
$\sum_{n=1}^{\infty} \frac{2^n}{n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n} \right)$		
$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$		
$\sum_{n=1}^{\infty} \cos(\pi n)$		
$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$		
$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{3n^3 + 1}$		
$\sum_{n=0}^{\infty} 3\left(-\frac{1}{2} \right)^n$		
$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$		
$\sum_{n=1}^{\infty} n e^{-n^3}$		
$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^n$		

$\sum_{n=1}^{\infty} \frac{1}{n^3+1}$		
$\sum_{n=0}^{\infty} \left(\frac{2}{9}\right)^n$		
$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$		
$\sum_{n=2}^{\infty} \frac{10n^2+n-2}{2n^6+7n-1}$		
$\sum_{n=1}^{\infty} \frac{n^2+3n-2}{\sqrt{4n^9+n-1}}$		