Math 3321 Homework 5

1. Find and classify the steady states of $y' = y - y^3$.
2. Find and classify the steady states of $y' = y^3 - y^2$.
3. Find and classify the steady states of $y' = \sin(y)$.
4. Find and classify the steady states of $y' = y - 10\cos(y)$ (you might need to use a root finder).
5. Approximate the solution to $y' = x - y\cos(xy),\ y(0) = 1$ on the interval [0,10]
    a. Using Euler's method with a step size of 0.01.
    b. Using Improved Euler's method with a step size of 0.01.
    (in each part, include a plot of the approximate solution and the approximate solution values at x = 0, 1, 2, 3, 4, … , 10)

6. Give the exact solution to the system

    $$u'(t) = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} u(t) + \begin{pmatrix} 1 \\ \cos(t) \end{pmatrix},\quad u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

    You will need this solution in problem 7.

7. Use the improved Euler's method with h = 0.01 to approximate the solution to

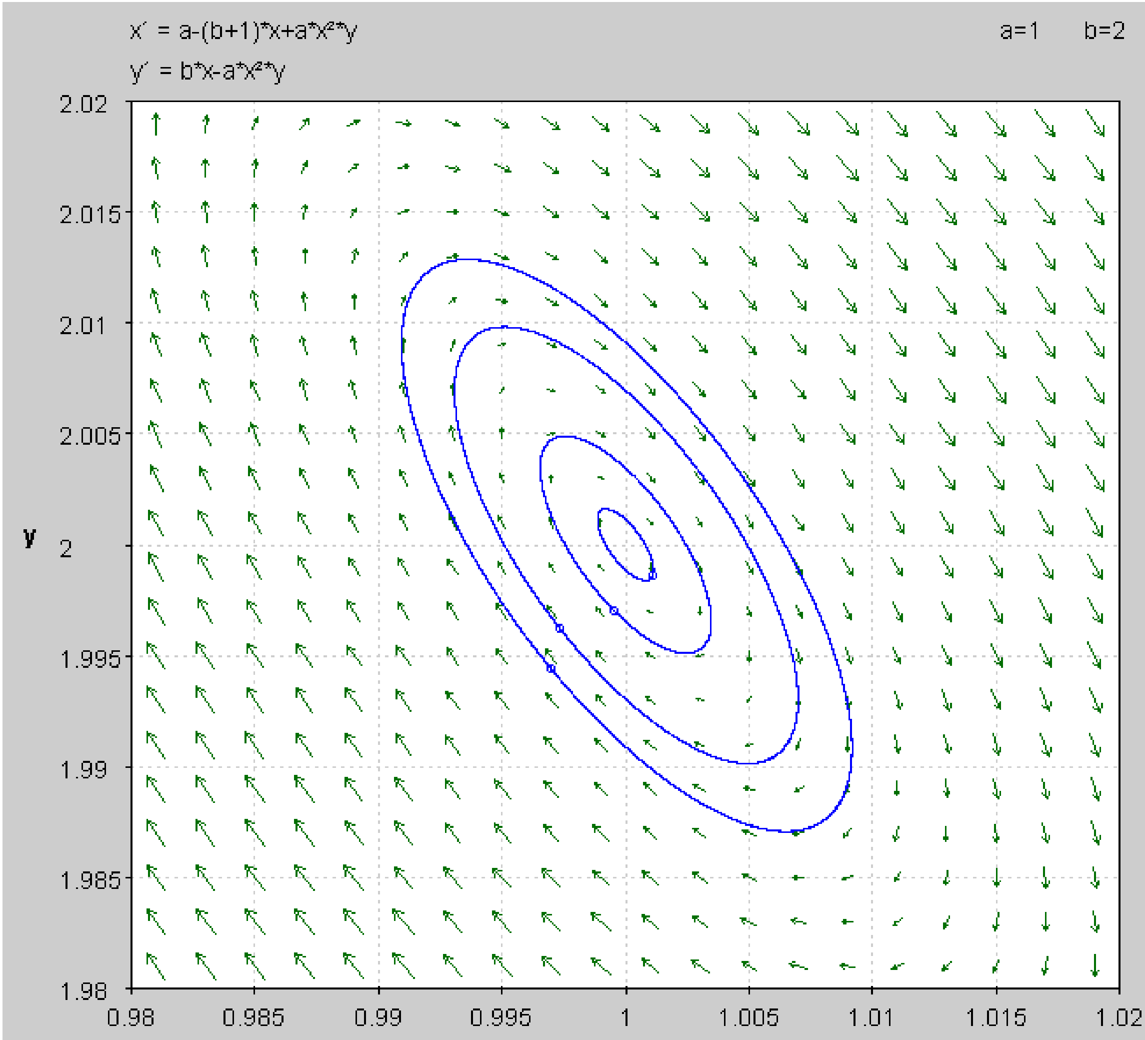
    $$u'(t) = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} u(t) + \begin{pmatrix} 1 \\ \cos(t) \end{pmatrix},\quad u(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

    over the interval [0,20]. Give a plot of the resulting solution components as functions of t. i.e. plot the first component of the approximate solution vs t, and then do the same for the second component. Also include the graph of the exact solutions for comparison. Finally, give a table that shows your approximations at k/5 for k = 0,…,100 along with the exact solutions. Be sure to document your work and explain how your results were obtained.

8. The Brusselator was discussed briefly in class, and the system is restated below.

    $$x' = a - (b+1)x + ax^2 y$$
    $$y' = bx - ax^2 y$$

    where a and b are positive constants and x and y are unknown functions of t. Positive initial data has to be supplied in order for the system to have a unique solution. If we set a = 1, b = 2 and provide initial data, then the solution ($x(t)$,$y(t)$) for $t > 0$ parameterizes a curve in the xy plane. I have shown 4 such solutions in the plot at the top of the next page.



Use improved Euler's method with a step size of 0.01 to attempt to approximate the solutions that are shown. In each case, give tables of data values and also a plot showing the parametric curves obtained from your approximate solutions. Be sure to document your work and explain how your results were obtained.

9. Find the steady states of each of the following systems, and classify each one as a sink, a source, a saddle, or neither.

   a. $$u' = u(4-u-v)$$ $$v' = v(5-u-2v)$$

   b. $$u' = u(9-3u-v)$$ $$v' = v(5-u-v)$$

   c. $$u' = 3v-u$$ $$v' = u-v^2$$

10. Create a phase plot for each of the systems in the problem above. Make sure your plots include sufficient information to see the behavior near all steady states of the systems. The software at http://math.rice.edu/~dfield/dfpp.html might be helpful.
11. Do a complete analysis of the behavior of steady states for the Brusselator depending upon the parameter values a and b.