Math 3321 Take-Home Midterm
Fall 2009

Directions: You may not talk to any person about the problems on this exam. Make a formal write-up of your findings/results and submit them in class on Monday, October 19^{th}.

Definition: A matrix is symmetric if and only if it is equal to its transpose.

1. Which of the following matrices are symmetric?
$$\begin{pmatrix} -1 & 2 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

2. Use the online matrix calculator to generate 50 random real 3 by 3 matrices and find the eigenvalues of each matrix. Do not list the matrices, but do report the number of matrices that have a complex eigenvalue.

3. Suppose a real 2 by 2 matrix is chosen at random, with integer entries in the set {-10, -9, -8, … , 8, 9, 10}. Determine the probability that the matrix will have complex eigenvalues.

4. Generate 50 random **symmetric** real 2 by 2 matrices. You can do this with the matrix calculator by generating a random 2 by 2 matrix and then changing the 2,1 entry to match the 1,2 entry. Calculate the eigenvalues for each of these matrices. How many of them have complex eigenvalues?

5. Repeat exercise 4 with 3 by 3 matrices, and report your findings.

6. Write a conjecture based upon your findings in problems 4 and 5, and prove that your conjecture is true for all symmetric real 2 by 2 matrices.

7. Generate 50 random real matrices of varying sizes. For each of these matrices, find the transpose of the matrix and then multiply the transpose of the matrix times the original matrix. Look carefully at the results. What do you observe?

8. Write a conjecture based upon your findings in problem 7, and prove that it is true.

9. Generate 50 random real matrices of varying sizes. For each of these matrices, find the transpose of the matrix and then multiply the transpose of the matrix times the original matrix. Find the eigenvalues of the resulting matrix. What do you observe?

10. Prove that if $A \in R^{m,n}, x \in R^n$ and $y \in R^m$, then $(Ax) \bullet y = x \bullet \left(A^T y\right)$.

11. Write a conjecture based upon your findings in problem 9, and prove that the conjecture is true. **Hint:** One way to prove your conjecture is to use problem 10.