Mathematical Neuroscience

Homework Set 2 - due 9/17/03

The following problems are taken from Wilson's book (with some modifications), which I have been following for the first two weeks of class. The notes I have handed out should be sufficient to do the homework, but you can consult the book if you get stuck. The book is on reserve at the Anderson Library

1. (prob. 2.3) At low light levels the parameter values of the retinal cones change and the feedback from the horizontal cells becomes weaker. A modified version of the equation we have used to model this system is given by

$$\frac{dC}{dt} = \frac{1}{\tau_C}(-C - 0.5H + L) \quad (1)$$

$$\frac{dH}{dt} = \frac{1}{\tau_H}(-H + C) \quad (2)$$

   where $\tau_C = 100$ ms and $\tau_H = 500$ ms. Obtain an exact solutions for different values of L. Plot the solutions using XPP.

2. Consider the system

$$E_1' = \frac{1}{\tau}(-E_1 + S(K_1 - 3E_2)) \quad (3)$$

$$E_2' = \frac{1}{\tau}(-E_2 + S(K_2 - 3E_1)) \quad (4)$$

   where E_1 and E_2 are two neurons in a network, S is the Naka-Rushton function, and K_1 and K_2 are inputs to the two neurons. What type of network is modeled by this equation, *i.e.* what type of influence do the two neurons have on each other?

   Given that $M = 100, N = 2$, and $\sigma = 120$ for the Naka-Rushton function $S(x)$, draw the nulclines of the system when $K_1 = K_2 = 120$. Find the fixed points and analyze their stability. This is called a *winner-take-all* (WTA) system. Explain why.

   A *basin of attraction* of a fixed point x_0 consists of all points which asymptotically approach x_0 in forward time. Using XPP try to guess what the basins of attraction are in this problem when $K_1 = K_2$, and where the boundary between these basins lies. This boundary is known as the *separatrix* (Hint: use symmetry).

   Extra credit: Describe in as much detail as you can what happens when $K_1 \neq K_2$.

3. (problem 8.1) The following is an equation that can be used to model action potential generation

$$V' = -V^3 + 3V^2 + 0.125 - R + I \quad (5)$$

$$R' = -R + 5V^2 - 1 \quad (6)$$

   If $I = 0$ plot all the fixed points and the nulclines. Draw possible locations for the limit cycles. Use XPP to check your results, and to see what happens as you increase the input current I.

4. The Wilson-Cowan equations we discussed in class originally incorporated a different non-linearity. An example is given by the following equation

$$E' = \frac{1}{5}\left(-E + \frac{100}{1+\exp(-0.1(1.6E - I + K - 40))}\right) \tag{7}$$

$$I' = \frac{1}{10}\left(-I + \frac{100}{1+\exp(-0.1(1.5E - 40))}\right) \tag{8}$$

- Analyze the nonlinearity on the right hand side and verify that it has a sigmoidal shape. Use graph sketching techniques you learned in calculus.
- Determine the stability characteristics of the steady state when $K = 0$.
- Show that there exists a Hopf bifurcation near $K = 10$.
- Simulate and plot results with a phase plane plot when $K = 15$. Use XPP to draw the nulclines in this case. You will need to learn about how to draw nulclines with XPP, which is discussed in the tutorial and the manual.