Mathematical Neuroscience

Homework Set 4 - due 10/27/03

1. Read pages 13–19 of Ermentrout's notes on passive membranes which you started reading for your last homework. Do the exercises on p. 16. Also perform an equivalent cylinder reduction to the example of a dendritic tree given on the bottom of figure 5 using the same method that is outlined in the notes.

2. Remember that Green's function is defined as the solution of the equation

$$-G'' + F = \delta(x - y) \qquad 0 < x < L, 0 < y < L$$

with appropriate boundary conditions at 0 and L.

   (a) Show that Green's function for a semi-infinite cable with boundary conditions $G'(0) = 0$ and $G(\infty) = 0$ has the form

$$G(x, y) = \begin{cases} e^{-y} \cosh x & \text{if } x < y \\ e^{-x} \cosh y & \text{if } y < x \end{cases}$$

   (b) Use this Green's function to compute the solution of the cable equation $-V'' + V = I$, where

$$I = \begin{cases} I_0 & \text{if } x_0 - \epsilon < x < x_0 + \epsilon \\ 0 & \text{otherwise} \end{cases}$$

   given that I_0 is some constant, and x_0, ϵ are positive constants such that $x_0 - \epsilon > 0$. The boundary conditions are the same as in the previous part.

3. After rescaling, the time dependent cable equation has the form

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} - V + I$$

where $I(x, t)$ is the input current density, and the boundary conditions are specified by

$$\alpha_1 V(0, t) + \beta_1 \frac{\partial V}{\partial x}(0, t) = 0 \qquad (1)$$

$$\alpha_2 V(L, t) + \beta_2 \frac{\partial V}{\partial x}(L, t) = 0 \qquad (2)$$

$$V(x, 0) = v_0(x)$$

Green's function $G(x, y, t)$ for this equation is the solution of

$$\frac{\partial G}{\partial t} = \frac{\partial^2 G}{\partial x^2} - G$$

with the same boundary conditions 1 – 2, and

$$G(x, y, 0) = \delta(x - y)$$

(a) Show that the solution of the time dependent cable equation can be expressed in terms of the related Green's function as

$$V(x,t) = \int_0^L G(x,y,t)v_0(y)dy + \int_o^L \int_0^t G(x,y,t-s)I(y,s)dsdy$$

You will need to make use of the following identity

$$\frac{\partial}{\partial t}\int_0^t f(s,t)ds = f(t,t) + \int_0^t \frac{\partial f(s,t)}{\partial t}ds$$

You need to check that both the equation and the boundary conditions are satisfied.

(b) Use this result and the two pages of notes from Tuckwell's book which are provided with this homework to obtain a solution for the time dependent cable equation with $G_x(0,y,t) = \lim_{L\to\infty} G_x(L,y,t) = 0$. If you understand the material given in the notes this will be a fairly simple calculation.

4. Read Bard Erementrout's notes entitled "Channeling with Bard". These are available at the bottom of the course webpage. Do the exercises on p. 9 of the notes using XPP. You will be able to find the equation in electronic form at

http://www.math.pitt.edu/ bard/bardware/classes/compneuro/model/models.html

so you do not need to type them in.

5. Use the ideas developed in the previous problem to encode the two dimensional reduction of the Hodgkin Huxley equation we discussed in class. We discussed two steps in the reduction

(a) Replace the equation for m' by assuming that $m(t) = m_\infty(V)$.

(b) Set $h = 1 - n = w$, as discussed in class.

After each step check whether the threshold I_0 you have found in the previous problem has changed. Check how each step, and the two steps combined, affect the shape of the action potential.