Mathematical Neuroscience

Homework Set 6 - due 12/12/03 - during finals

1. Read the following link on stochastic models of channels:

   http://www.math.pitt.edu/˜bard/bardware/classes/compneuro/stoch/stoch.html

   Two XPP files are included here. Look through the files and make sure you understand what the simulations are designed to do. Do the homework at the end of this page.

   Here is a very basic explanation Markov chains and the transition matrix which are what these simulations are based on:

   http://www.aw-bc.com/greenwell/markov.pdf

2. In class I outlined the derivation of the Fokker-Planck equation. I followed the derivation on pages 178 – 180 in the chapter by Gerstner and Kistler which is available on my webpage. There are several steps in the mathematical derivation which I have skipped. Start with equation (5.84).

   (a) Explain briefly how equation (5.85) is derived. Also, give an interpretation of equation (5.86). Think about a distribution $p(u', t)$ that is concentrated at a finite number of points, so that this integral becomes a sum. What does this integral mean in this case.

   (b) Complete the derivation of equation (5.89), filling in all the details that are left out in the text. This will involve taking some Taylor expansions, and limits.

   (c) Suppose that we have a neuron that is bombarded by a Poisson input, as in the previous example. Suppose that we have two Poisson processes (modeling two groups of presynaptic cells), such that their rates of firing are equal and independent of time, $\nu_1 = \nu_2$. Also assume that their synaptic weights are the same, but of opposite sign. What is the resulting Fokker-Planck equation? This should follow immediately simply by replacing several of the constants.

3. Prove that the Fokker-Planck equation without the drift term has a solution of the form $u(x,t) = \delta(x - x_S(t))$, where $x_S(t)$ is a solution of a certain ordinary differential equation. You should be able to do this with the information given in class.

4. A Poisson process can also be simulated using XPP, see

   http://www.math.pitt.edu/ bard/bardware/classes/compneuro/poisson/poisson.html

   Create a simple integrate and fire neuron and use a spike train of δ function generated by a Poisson process in this way as an input. Simply use an additive input, so that the delta spikes cause an approximately instantaneous increase in membrane voltage.

   What happens as you increase the synaptic weight, given that the input is excitatory? Change the input current so that, without noise, the system is just below threshold. What happens as you add a little noise, and then keep increasing it?

   Finally, illustrate the phenomenon of stochastic resonance. Take an input current of the form $I_{app} = I_0 + A\sin(\Omega t)$ where A is not too large, and the signal is chosen in such a way that without the input the neuron never passes threshold. If you add too little noise, you should see that the system spikes very rarely. If you add just the right amount of noise, the spiking should be close to periodic (exactly how this will work will depend on the relation between your membrane constant τ_m and Ω, so keep track of the relation between the two). Finally, if the input is too strong, the approximate periodicity should disappear.

Here is an interesting short article on how noise can enhance balance control and sensitivity, based on a very similar idea

http://bme.bu.edu/faculty/Documents/lancet.pdf