Mathematical Neuroscience

Homework Set 1 - due 2/11/04

1. You will find two papers that address the question "What makes a neuron spike?" on the course webpage, under Additional Resources. The first paper is by Gray and Azouz, and the second by Mainen and Sejnowski. Give a short outline of these papers, and try to pay attention to the following questions:

   Gray and Azouz: How was the threshold calculated, and does this make sense keeping in mind what we learned last semester? One of their conclusion is that cells are sensitive to synchronous synaptic input – how do they quantify and justify this statement in the paper? What are the potential problems with the approach they use? How do the numerics compare to the data?

   Mainen and Sejnowski: From their argument, do you think that a low pass filtered white noise stimulus is best in obtaining a reliable response? If so can you speculate why, and if not, what other types of stimuli do you think would yield similar results? What could these results imply about neurons in networks where a single neuron cannot be isolated, and where the input is not the same from trial to trial?

2. a) During the last semester you already took a quick look at Bard Ermentrout's program for generating a Poisson spike train at

   http://www.math.pitt.edu/~bard/bardware/classes/compneuro/poisson/poisson.html

   Use this program to verify numerically that 1) the distribution $P_T[n]$ as a function of n really approaches a Gaussian function, 2) The spike train really becomes more regular as t_{ref}, the refractory period is increased. 3) Show a drop in the coefficient of variation as t_{ref} is increased, while other parameters are held constant.

   b) Do a couple examples of spike triggered averaging, as discussed here

   http://www.math.pitt.edu/~bard/bardware/classes/compneuro/sta.html

   If you are having trouble getting the files, let me know and I will send them to you.

3. Compute the first two moments of the Poisson distribution $P_T[n]$:

$$
\begin{aligned}
< n > &= \sum_{n=0}^{\infty} n P_T[n] \\
\sigma_n^2(T) &= \sum_{n=0}^{\infty} n^2 P_T[n] - < n >^2
\end{aligned}
$$

   where $P_T[n] = ((rT)^n/n!)\exp(-rT)$ as shown in class.

   Hint: Use the moment generating function

$$
g(\alpha) = \sum_{n=0}^{\infty} \frac{(rT)^n \exp(\alpha n)}{n!} \exp(-rT)
$$

and show that $\frac{dg}{d\alpha}|_{\alpha=0} =< n >$ and $\frac{d^2g}{d\alpha^2}|_{\alpha=0} = \sum_{n=0}^{\infty} n^2 P_T[n]$. Compute these derivatives remembering the definition $\exp(x) = \sum_{n=0}^{\infty} x^n/n!$, and evaluate them at 0. You should get rT for both moments, as discussed in class. If you get stuck look in Appendix B of Dayan and Abbott for more hints.

4. (*) Derive the probability density for n spike times for an inhomogeneous Poisson process

$$p[t_1, \ldots, t_n] = \exp\left(-\int_0^T r(t)dt\right) \Pi_{i=1}^n r(t_i).$$

Start by computing the probability that no spike occurs between the times t_i and t_{i+1} by dividing this interval into bins and using the same argument we used in class. You should get a product with terms depending on Δt. To evaluate this expression take the logarithm to turn it into a sum and the approximation $\ln(1-\delta) \approx -\delta$ which is valid for small δ. You should get a sum which in the limit $\Delta t \to 0$ turns into the integral

$$-\int_{t_i}^{t_{i+1}} r(t)dt.$$

You will need to exponentiate this to get rid of the ln introduced to generate the sum. Now multiply this by the probability of generating the spikes during the given times, and rearrange terms.

Again, for more hints see the Appendix to Chapter 1 in Dayan and Abbott.