Mathematical Neuroscience

Homework Set 6 - due 5/10/04 at 5pm (final exam)

Problems that are marked with a (*) are required only for graduate students in the class. Problems marked with (**) are required only for undergraduates (although graduate students should consider doing these as exercises anyway). Unmarked problems are required for everybody.

1. Read sections 1,2,3,5,6, and 7 of the paper entitled "Neural Network as Spatio-Temporal Pattern Forming Systems" by Bard Ermentrout (nnetrev.pdf). Discuss which properties of the individual cells are incorporated in the neural networks discussed here, and which are ignored. What is meant by pattern formation? Briefly discuss the mathematical tools that are proposed for the analysis of the activity in such networks.

2. Consider the McKean model of a relaxation oscillator

$$\begin{aligned} \epsilon v' &= f(v) - w - w_0 + I(t) && (1) \\ w' &= v - \gamma w - v_0, && (2) \end{aligned}$$

   where

$$f(v) = \begin{cases} -v, & v < \alpha/2 \\ v - \alpha, & \alpha/2 < v < (1+\alpha)/2 \\ 1 - \alpha, & v > (1+\alpha)/2 \end{cases} .$$

   For the purposes of this exercise you can set $\alpha = 0$, and let $I = 0$ at first. Show that the nullclines of this model agree with what is shown in Fig. 1 of the paper given in the file *coombes.pdf*. Choose the remaining constants so that the nullcline determined by $w' = 0$ intersects the left branch of the "cubic" nullcline in the way shown (the precise values will not be important).

   Is this an excitable system, that is, if sufficient constant current $I(t) = I_0$ is injected, can the system respond with a spike? What needs to be true for this to be the case?

   Suppose that the system receives a periodic train of excitatory impulses, to which it responds instantaneously (you may think of $I(t)$ as composed of a periodic sequence of square pulses). Derive the Poincaré Map P for this periodic input in the singular limit $\epsilon \to 0$, as follows:

   The domain of the map are the values v of the points on the left hand branch. $P(v)$ is the value of v after one period of the input. Under appropriate conditions of the period, you should obtain a figure similar to the one in Fig. 2 in *coombes.pdf*.

3. Explore the associative network discussed in class and given in

   `http://www.math.pitt.edu/~bard/bardware/classes/compneuro/assoc.ode`

   Can you recreate the different memories in this system? Use the ARRAY plot as described to see all the values.

4. (*) he following file illustrates Hebbian learning with two weights.

   `http://www.math.pitt.edu/~bard/bardware/classes/compneuro/rlrn.ode`

   Explore the model numerically. Describe what the equations do, and discuss whether the numerics agree with your expectations.