Many experiments have found that both humans and animals showed history-dependent biases. For instance, Hwang et al. found that mice had predicted a choice prior to the subsequent trials. Goldfarb et al. found that humans reacted faster when repeated states were more likely. In decision-making study, feedback helps decision makers adjust their behaviors on subsequent trials. Human subjects are able to adjust their strategies based on performance feedback: positive feedback (negative feedback) encourages subjects to make the same decision (switch to opposite decision) on repeated trials. In animal studies, reward is given as feedback. Bromberg-Martin et al. found that probabilistic reward motivated animals perform tasks. Most mathematical models focus on idealized situation while some explain the behaviors well but do not suggest decision strategies. Here, we considered normative probabilistic models of ideal observers who evaluate and maximize their expected rewards to select the best decisions. Normative models help understand the nature of decision-making processes better. In this dissertation, we focus on two factors of a more realistic decision-making model: history-dependent biases and probabilistic feedback.

We first model an ideal observer in a sequence of trials in which correct choice is switched between two options. We assume that no performance feedback is given at the end of each trial because we are interested in the observer's decision strategy in the absence of feedback. The obsever computes their internal belief to determine which choice is more likely correct. On subsequent trials, the observer biases their belief on subsequent trials based on previous decision and on how likely the true choice is switched. We quantify this bias as an initial belief. If the choice is more likely to repeat, the observer has a bias toward their last decision. Otherwise, the observer will discount their past beliefs as they are no longer relevant. Taking time to accumulate information, the observer will be more accurate. However, spending too much time on one trial will decrease the performance overall as the observer will have less time for future trials. We conclude that the best strategy to obtain the most rewards is to spend more time on earlier trials to increase the bias and make quicker decisions on later trials.

We next consider how the observer integrates external feedback with their internal belief on the first two trials. Feedback is then given in two possible ways. *Probabilistic signaling* denotes feedback given after the response as a signal: green (red) light implying the response was more likely correct (incorrect). The probabilistic light signal enhances the observer's belief as it gives more information about the correct choice. When feedback is reliable, the observer's belief is strong so they can make immediate decisions. Alternatively, the reward itself may be provided at the end of each trial, but with some probability of being received when the observer is wrong, or not received when the observer is right. We call this the case of *probabilistic reward.* In this case, the observer prioritizes getting reward rather than being correct. When probability getting reward for correct choice is low, or feedback is unreliable, the optimal strategy is making immediate decision to minimize the decision time as the accuracy is not important. Therefore, probabilistic signaling improves the observer's accuracy while probabilistic reward improves their decision time.

Our model describes the behavior of an ideal observer that can be used to determine experimental subjects' strategies. Furthermore, our normative model opens a window to more complex state-histories on the evidence-accumulation strategies. Uncovering common assumptions about state-histories will help guide future experiments, and help us better quantify the biases and core mechanisms of human decision-making.