## Instructions

This package of code is meant to calculate correlations via Monte Carlo simulation and linear response theory for a network of EIF neurons with membrane potentials (mV) evolving according to

$$\tau \dot{v}_i(t) = -v_i(t) + \psi(v_i(t)) + E_0 + \sqrt{\sigma^2 \tau} \xi_i(t) + f_i(t).$$

 $\tau = C_m/g_L$  (ms) is the membrane time constant,  $C_m, g_L$  are the membrane capacitance and conductance,  $E_0$  (mV) the effective rest potential,  $\sigma$  (mV) is the noise variance,  $\xi_i(t)$  is an independentlyevolving white noise process, and  $f_i(t)$  captures synaptic coupling between the neurons. The function  $\psi$  takes the form

$$\psi(v) = \Delta_T \exp\left[\frac{v - v_T}{\Delta_T}\right]$$

where  $v_T$  (mV) sets the soft threshold of the EIF model, and  $\Delta_T$  (mV) is a spike shape parameter. The synaptic coupling is, by default, through alpha-shaped kernels, so that

$$f_i(t) = \sum_j (\mathbf{J}_{ij} * y_j)(t), \quad \text{where} \quad \mathbf{J}_{ij}(t) = \begin{cases} \mathbf{W}_{ij} \left(\frac{t - \tau_{D,j}}{\tau_{S,j}^2}\right) \exp\left[-\frac{t - \tau_{D,j}}{\tau_{S,j}}\right] & t \ge \tau_{D,j} \\ 0 & t < \tau_{D,j} \end{cases}.$$

where  $g_L \mathbf{W}_{ij}$  sets the total area under the post-synaptic current evoked in cell *i* by a spike in cell *j*, and  $\tau_{S,j}, \tau_{D,j}$  are synaptic time constants and delays for the output of cell *j*.

## Simulations

The simulations are set up to run for a 2 cell, unidirectional excitatory network by default. The parameter files are expected to be ASCII. In particular, the connection matrix (conn\_mat.dat) should be an  $N \times N$  matrix with weights in units (mV ms), and the synaptic time constants (syn\_time\_consts.dat) and delays (syn\_delays.dat) should be  $N \times 1$  values in units (ms). All other required parameters are listed in the file "sim\_caller.cpp", which should be compiled and run to execute the simulations.

Note that, by default, the simulations are set to place output in the subfolder "output/", which should be created before attempting to compile the code. The output location can be changed in the simulation library file, "EIF\_Cu\_AS\_AN.h". The simulation takes the parameters listed in the aforementioned caller program, along with the mentioned files, and returns cross-correlations for pairs listed in "ccg\_inds.dat", which should begin with a number indicating the number of pairs correlations will be calculated for, along with a list of this many pairs (see include file for example format). The simulation also returns as output the stationary firing rates, CVs of the interspike interval distribution, and the simulation time in seconds.

## Linear response theory

The linear response theory takes all the same parameters as the simulations. The code to solve for the linear response is written in MEX within MATLAB, and the included caller file "LR\_caller.m" is a MATLAB script. Four main files are included

- calc\_Rate\_cuEIF.cpp Calculates the stationary rate of an uncoupled EIF.
- calc\_Power\_cuEIF.cpp Calculates the power spectrum of an uncoupled EIF at a list of frequency values.
- calc\_Susc\_cuEIF.cpp Calculates the susceptibility (Fourier transform of the linear response function) of an uncoupled EIF at a list of frequency values.
- LR\_caller.m Shows how to use the linear response theory to calculate correlations.

We have also included some convenient code for calculating inverse Fourier transforms which is used to retrieve cross-correlations once the cross-spectra are solved for using linear response theory.

Following the methods in our paper, solving for cross-correlations using linear response theory proceeds in four clearly marked steps:

- 1. Calculate the rates in the recurrent network via a simple fixed-point iteration using the included code.
- 2. Use the included code to calculate uncoupled power spectra and response functions for cells in the network, along with the frequency-domain representations of the synaptic kernels.
- 3. Solve the matrix equations for the linear response approximations to the auto- and cross-spectra in the network.
- 4. Take the inverse Fourier transform of the spectra to obtain the linear response predicted autoand cross-correlations.