

Name:

### Test 3      Math 3336

This test is worth **200** points, **20** points for each problem.

You have **50** minutes to complete the test. You cannot use any books or notes.

1. Prove by induction that 3 divides  $n^3 + 2n$  whenever  $n$  is a nonnegative integer.
2. Show that if an integer is divisible by 3 if and only if the sum of its decimal digits is divisible by 3.
3. (a) Define that  $R$  is an equivalence relation on the set  $A$ .  
 (b) Define that  $\pi$  is a partition of the set  $A$ .  
 (c) Describe the partition  $\pi_R$  for an equivalence  $R$ .  
 (d) Define the equivalence  $R_\pi$  for a partition  $\pi$ .
4. Determine which of the following relations on the set  $\mathbb{Z}$  of integers are equivalence relations.
  - (a)  $aRb$  iff  $a - b \geq 0$ .
  - (b)  $aRb$  iff  $ab \geq 0$ .
  - (c)  $aRb$  iff  $|a| = |b|$ .
  - (d)  $aRb$  iff  $|a - b| \leq 1$ .

5. Let  $E$  and  $F$  be equivalence relations on the set  $A = \{a, b, c, d, e\}$  where the partition for  $E$  is given by

$$\pi_E = \{\{a\}, \{b, c, d\}, \{e\}\}$$

and the partition for  $F$  is

$$\pi_F = \{\{a\}, \{b, d\}, \{c, e\}\}$$

Find the partition of  $E \vee F$  and  $E \wedge F$ .