1. Prove that a finite abelian group that is not cyclic contains a subgroup which is isomorphic to $\mathbb{Z}_p \oplus \mathbb{Z}_p$.

2. Let $A$ be an abelian group of order $n$ and let $m$ be an integer that divides $n$. Prove that $A$ contains a subgroup of order $m$.

3. A finite abelian group is directly irreducible if it is not the direct sum of abelian groups of smaller order. Give a direct proof that every finite abelian group is the direct sum of directly irreducible abelian groups. Characterize the directly irreducible abelian groups.

4. Let $A, B, C$ be finite abelian groups. Show that if $A \oplus B \cong A \oplus C$ then $B \cong C$.

5. Let $n$ be a positive integer. Show that every abelian group of order $n$ is cyclic iff $n$ is not divisible by the square of any prime.