Section 1.6
Systems of Measurement

Measuring

Systems of numeration alone do not provide enough information to describe all the physical characteristics of objects. With numerals, we can write down how many fish we caught on our weekend fishing trip. However, numerals alone cannot adequately describe how big the fish were, or how long was the longest fish. To answer these questions we also need words and symbols for measurement.

To measure a physical attribute of an object is to compare its magnitude with some standard. We measure length by comparing the length of an object to the length of our ruler or yardstick. The standards for measuring a particular quantity are called units of measurement. Once a standard is agreed upon, the magnitude of the quantity to be measured for a particular object is given in proportion to the standard.

Definition: A **system of measurement** is a set of units which can be used to specify any physical attribute of an object.

There are two systems of measurement used in the United States today. The first is the US Customary System, also sometimes referred to as the English System. This system has its roots in the system used in the British Empire in colonial days. The units in the US Customary System includes the common measuring units used every day to measure length in feet and inches, distances in miles, weight in pounds and ounces, and temperature in degrees Fahrenheit. The disadvantage of the US Customary System is that more than one basic unit is used to measure length, weight, and volume. Furthermore, the relationships between these basic units are not necessarily consistent. For example, there are 12 inches in a foot, 3 feet in a yard, 5280 feet in a mile, etc. Therefore, converting length measurements from feet to miles, or inches to yards requires a significant amount of arithmetic.

The second system of measurement used in the United States is the International System of Units (SI), also called the metric system. The SI system is the universally accepted system for science and trade and commonly used in most of the rest of the world. This system adopts ONE basic standard of measurement for each property. Other convenient units are derived by multiplying the base unit by powers of ten. Thus, the metric system is “measurement meets a base ten place value system”.

Basic Units

Scientists generally agree that there are seven fundamental quantities to measure. They are length, mass, time, temperature, electrical current, luminous intensity and amount of a substance (number of atoms or molecules). All other quantities, like area and volume, are derived from the fundamental quantities. In this section we will look at the fundamental quantities of length, mass and temperature and also the derived quantities of area and volume. Common units for time are understood by most students and need no further explanation. The quantities of electrical current, luminous intensity and amount of a substance are more appropriately addressed in science class.

The mass of an object is a measure of amount of matter present in an object. The weight of an object refers to the force exerted by gravity on that object. The terms “weight” and “mass” are often used interchangeably, since the force exerted by earth’s gravity on an object is proportional to its mass. However, they are not the same quantities. The mass of an astronaut on earth is the same as his mass on the moon. Yet his weight is much less on the moon, since the moon’s gravity is only about one-sixth of that of the earth. Thus, the quantity most students normally associate as weight should more appropriately be called mass, and units for mass will be discussed in this text.

Basic Units in the US Customary System

The following chart lists the basic units for length, mass and volume in the US Customary System. Note that the relationships between the various basic units for any quantity are fairly complicated!

1 foot (ft)	12 inches (in)
1 yard (yd)	3 feet
1 mile (mi)	5280 feet
1 pound (lb)	16 ounces
1 ton (T)	2000 pounds
1 cup (c)	8 fluid ounces
1 pint (pt)	2 cups
1 quart (qt)	2 pints
1 gallon (gal)	4 quarts

The basic unit of temperature in the US Customary System is degrees Fahrenheit. On the Fahrenheit temperature scale, water freezes at $32^{\circ}F$ and water boils (at sea level on the earth) at $212^{\circ}F$.

Basic Units in the Metric System

The basic unit for length in the metric system is the meter. It was historically defined as one ten-millionth of the length of the meridian through Paris from the north pole to the equator. The length was calculated and a prototype was made in metal. Since then, the length has been defined much more precisely by physicists. However, the basic idea is still to have one “official standard meter” kept securely under constant conditions and compare all other lengths to this standard.

Although volume is derived from length, the metric system contains a standard unit for volume, called the liter. It is defined to be the volume of a cube that has sides of length one-tenth of a meter. This is a convenient unit for measuring everything from quantities of milk to gasoline. Volume is also commonly measured in cubic length units, since volume is very often calculated by a formula involving the different dimensions of an object.

The base unit for mass is a gram. Historically, the basic unit for mass in the metric system is a kilogram, originally defined to be the mass of one cubic decimeter of water. The gram is defined to be one-thousandth of a kilogram. However, to keep our discussion of units, prefixes and conversions consistent, the gram will be considered the base unit for mass in this text.

The base unit for temperature in the metric system is degrees Celsius. On the Celsius temperature scale, water freezes at $0^{\circ}C$ and water boils (at sea level on the earth) at $100^{\circ}C$.

Metric Prefixes

The metric system creates measuring units of varying magnitudes by adding prefixes to the base units. Each prefix represents multiplying the size of the base unit by a different power of ten. The prefixes are listed in the table below.

Multiple of the Base	**Prefix**	**Symbol**
$10^{12} = 1{,}000{,}000{,}000{,}000$	tera -	T
$10^{9} = 1{,}000{,}000{,}000$	giga -	G
$10^{6} = 1{,}000{,}000$	mega -	M
1000	kilo -	k
100	hecto -	h
10	deka -	da
$10^{-1} = \frac{1}{10}$	deci -	d
$10^{-2} = \frac{1}{100}$	centi -	c
$10^{-3} = \frac{1}{1000}$	milli -	m
$10^{-6} = \frac{1}{1000000}$	micro -	μ
$10^{-9} = \frac{1}{1000000000}$	nano -	n
$10^{-12} = \frac{1}{1000000000000}$	pico -	p

The prefixes for multiplying and dividing the base unit by up to 10^{12} are listed in the table. However, the exercise material in the text only requires familiarity with multiples from one-thousandth through one thousand times the base. The prefixes for very large and very small units are listed because these prefixes occur in our everyday lives. For example, when a computer manufacturer claims the memory of his machine is measured in gigabytes, in is important to know that a gigabyte is one thousand times larger than a megabyte. Or, when reading information on nanotechnology, the prefix "nano-" should easily be understood to be "very small scale".

Here are the metric prefixes and the relationship between the different units of length. This same chart can be used for mass or volume if the "m" for meter is replaced with an "l" for liter (volume) or a "g" for gram (mass).

Prefix	Symbol	Size
kilo	k	1 km = 1000 m
hecto	h	1 hm = 100 m
deka	da	1 dam = 10 m
-	-	1 m
deci	d	1 dm = 1/10 m
centi	c	1 cm = 1/100 m
milli	m	1 mm = 1/1000 m

You can remember the prefixes using the sentence "King Henry Danced Merrily Down Center Main", understanding that the M stands for meter. You'd need to change "meter" to gram or liter if you were not measuring length.

Converting Between Metric Units

To measure the length of an object, the procedure is to choose a standard unit to compare the object's length to. Which unit should be chosen? There is not a single correct answer to the question of which unit to choose. The best unit to choose may depend on how accurately the measurement needs to be made or a desire to compare the object's length against another length in a specific unit. Therefore, it is critically important to be able to convert a length measured in one metric unit to the appropriate measurement in another unit. Similarly, it is important to be able to convert mass and volume measured in one metric unit to the corresponding measure in a different unit.

Fortunately, the base ten properties of the metric system make converting measurements in one unit to the corresponding measurement in another unit a simple matter of multiplying or dividing by a power of ten. Since the place values of Hindu-Arabic numbers are powers of ten, multiplying or dividing by a power of ten is simply performed by moving the decimal point.

The following table is a useful illustration of the procedure for converting metric units. The first row of the table lists the metric prefixes, and the second row shows the length of one meter in each of the metric units. Notice that the same length is **DIVIDED** by ten when we move from a smaller unit to next larger unit and **MULTIPLIED** by ten when we move from a larger unit to a smaller unit.

Equivalent Measures for 1 Meter

Unit	Kilometer	Hectometer	Decameter	Meter	Decimeter	Centimeter	Millimeter
Symbol	km	hm	dam	m	dm	cm	mm
1 m =	0.001 km	0.01 hm	0.1 dam	1 m	10 dm	100 cm	1000 mm

For any metric measurement, whether length, mass or volume, to convert from one unit to a different size unit:

- To move from a smaller unit to a larger unit, move the decimal point one place to the left for each larger unit of measurement until you get to the one you need.

- To move from a larger unit to a smaller unit, move the decimal point one place to the right for each smaller unit of measurement until you get to the one you need.

Example 1: Convert the following length measurements to the indicated units.

a. 2.34 m = ________ cm
b. 4013 mm = ______ dm
c. 0.985 km = _______ m
d. 69.45 hm = ______dam
e. 0.0361 m = ______ mm
f. 0.23 mm = _______ cm
g. 340 km = ________ cm
h. 4.003 dm = _______ hm

Solution:

a. Centimeters are 2 places to the right of meters on the Equivalent Measures for 1 Meter table. This means that one centimeter is two powers of 10 SMALLER than one meter. This means to convert from meters to centimeters, the numerical value of the measurement must be MULTIPLIED by 100. To accomplish this, move the decimal place two places to the right.

$$2.34m = \underset{\to 2}{234}\,cm$$

b. Decimeters are two places to the left of millimeters on the Equivalent Measure for 1 Meter table. This means that one decimeter is two powers of ten LARGER than one millimeter. To convert from millimeters to decimeters, the numerical value must be DIVIDED by 100. To accomplish this, move the decimal point 2 places to the left.

$$4013mm = 40.\underset{2\leftarrow}{13}dm$$

c. 0.985 km = 985 m
d. 69.45 hm = 694.5 dam
e. 0.0361 m = 36.1 mm
f. 0.23 mm = 0.023 cm
g. 340 km = 34000000 cm
h. 4.003 dm = 0.004003 hm

Example 2: Convert the following mass measurements to the indicated units.

a. 31.234 g = ________kg
b. 0.0024 kg = _______cg
c. 45001 mg = _______kg
d. 0.351 dg = ________ hg
e. 3005 mg = ________ g
f. 431.52 hg = _______ cg
g. 0.678 mg = ________ g
h. 0.003 g = __________ kg

Solution

The Equivalent Measures for 1 Meter chart can also be used to convert mass units by replacing "meter" with "gram". The procedure for multiplying or dividing by powers of ten by moving the decimal place works with exactly the same formula.

a. 31.234 g = 0.031234 kg
b. 0.0024 kg = 240 cg
c. 45001 mg = 0.045001 kg
d. 0.351 dg = 0.000351 hg
e. 3005 mg = 300.5 g
f. 431.52 hg = 4315200 cg
g. 0.678 mg = 0.000678 g
h. 0.003 g = 0.000003 kg

Example 3: Convert the following volume measurements to the indicated units.

a. 4301 l = ___________ kl
b. 34.8 ml = __________ l
c. 9.009 cl = __________ ml
d. 7.5 dal = ___________ l
e. 8 l = _______________ ml
f. 0.0406 l = ___________ cl
g. 0.000004 l = _________ ml
h. 1 kl = _______________ ml

Solution:

The procedure for volume units works the same as that for length and mass.

a. 4301 l = 430.1 kl
b. 34.8 ml = 0.0348 l
c. 9.009 cl = 90.09 ml
d. 7.5 dal = 0.75 l
e. 8 l = 8000 ml
f. 0.0406 l = 4.061 cl
g. 0.000004 l = 0.004 ml
h. 1 kl = 1000000 ml

Appropriate Units for Common Measurements

A basic understanding of the metric system includes an intuitive understanding the size of each metric unit. The following table shows the rough relationship between the units in the metric system and similar units in the US Customary System.

1 meter	A little longer than 3 feet (1 yard)
1 kilogram	A little more than 2 pounds
1 liter	A little more than a quart
1 cm	About one-half an inch
1 km	About six-tenths of a mile
1 gram	1 ounce is about 28 grams

These approximate

Example 4: Which metric length unit (kilometer, meter, centimeter or millimeter) should you use to measure each of the following distances

a. your height
b. the length of the floor of the classroom
c. the width of a human hair
d. the length of your foot
e. the distance from Houston to Austin

Solution:

a. meter
b. meter
c. millimeter
d. centimeter
e. kilometer

Example 5: Which metric mass unit (kilogram, gram or milligram) is the most appropriate to measure the mass of each of the following objects?

a. your mass
b. the amount of salt to add to a cookie recipe.
c. the mass of the oranges you are buying at the grocery store
d. the mass of a paper clip
e. the mass of a dose of medicine.

Solution:

a. kilograms
b. grams
c. kilograms
d. grams
e. milligrams

Example 6: Which metric volume unit (kiloliter, liter, deciliter or milliliter) is most appropriate to measure the following quantities?

a. the amount of water you need to put in a pot to make some rice
b. the amount of milk in a carton of milk
c. the amount of cough medicine to give a young child
d. the amount of orange juice in a small glass
e. the amount of water in a swimming pool

Solution:

a. liter
b. liter
c. milliliter
d. deciliter
e. kiloliter

Converting Temperature Between Degrees Fahrenheit and Degrees Celsius

In the US Customary System, temperature is measured in degrees Fahrenheit. In the metric system, temperature is measured in degrees Celsius. The following formulas provide the way to translate between these two scales. In the formulas, the variable "C" stands for the temperature in degrees Celsius, and the variable "F" stands for the temperature in degrees Fahrenheit.

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

Example 7: Use the temperature conversion formulas to convert the following temperatures in degrees Fahrenheit into degrees Celsius.

a. $80^\circ F$
b. $40^\circ F$
c. $10^\circ F$
d. $350^\circ F$

Solution:

The appropriate Celsius temperature is found by using the formula $C = \frac{5}{9}\left(F - 32\right)$.

a. $C = \frac{5}{9}\left(80 - 32\right) = 26.7$

b. $C = \frac{5}{9}(40-32) = 4.4$

c. $C = \frac{5}{9}(10-32) = -12.2$

d. $C = \frac{5}{9}(350-32) = 176.7$

The Celsius temperatures, rounded to one decimal place, are

a. $80^\circ F = 26.7^\circ C$
b. $40^\circ F = 4.4^\circ C$
c. $10^\circ F = -12.2^\circ C$
d. $350^\circ F = 176.7^\circ C$

Example 8: Convert the following Celsius temperatures into temperatures in degrees Fehrenheit.

a. $50^\circ C$
b. $25^\circ C$
c. $200^\circ C$
d. $-10^\circ C$

Solution:

The Fahrenheit temperature is found by using the formula $F = \frac{9}{5}C + 32$.

a. $F = \frac{9}{5}(50) + 32 = 122$

b. $F = \frac{9}{5}(25) + 32 = 77$

c. $F = \frac{9}{5}(200) + 32 = 392$

d. $F = \frac{9}{5}(-10) + 32 = 14$

The equivalent temperatures are

a. $50^\circ C = 122^\circ F$
b. $25^\circ C = 77^\circ F$
c. $200^\circ C = 392^\circ F$
d. $-10^\circ C = 14^\circ F$

Examples 7 and 8, combined with the table below on the freezing and boiling point of water should help us learn to estimate temperature measured in degrees Celsius.

Celsius	Meaning	Fahrenheit
0°C	Freezing point of water	32°F
100°F	Boiling point of water	212°F

The following example tests our ability to recognize the appropriate Celsius temperatures associated with every day life.

Example 9: Match the item in the first column with an appropriate temperature in the second column.

A. A cup of hot coffee	1. 5°C
B. The air temperature on a summer day in Houston	2. 90°C
C. The temperature in your refrigerator	3. 175°C
D. The temperature for baking cookies	4. 33°C
E. The temperature of bathwater	5. -15°C
F. The temperature of in Buffalo, NY, on January 1	6. 40°C

Solution:

1. C 2. A 3. D 4. B 5. F 6. E

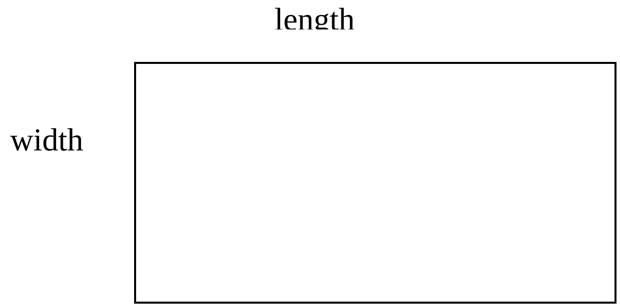
Measuring Area and Volume by Formulas

Area

The units for area of a two dimensional region are square length units. A square foot is the area of a square that has sides of length one foot. A square meter is the area of a square with sides of length one meter. In the US Customary System, area is often measured in square inches, square feet or square miles. In the metric system, area is measured in square meters. Using the metric prefixes, area can also be measured in square kilometers, square centimeters, etc. The notation for a square length unit is the unit symbol with a "2" superscript on the abbreviation for the unit. Thus 4 square meters would is written 4 m^2. Also, the proper notation is to write the numerical value, then a space and then the unit abbreviation.

It is very important to realize that a square centimeter is NOT one-hundredth the size of a square meter. Each side of the square meter is 100 times longer than the side of the square centimeter. So, when you calculate the area of the square meter, it is 100 * 100, or 10,000 times bigger than the area of the square centimeter!

The area of a rectangle is calculated by the formula:



Area = Length x Width

To use this formula to calculate the area of a rectangular (or square) region in the plane in a square metric unit:

1. Convert the width and length of the rectangle to the appropriate unit.
2. Multiply the width times the length and express your answer in square units.

Example 10: Find the area of a rectangle, in square centimeters, that is 3 cm long and 1.5 mm wide.

Solution:

The answer is in square centimeters, so convert the width of the rectangle to centimeters by dividing by 10 (We are going from a smaller unit to the next larger unit).

Width = $1.5 \div 10 = 0.15$ cm.

Then multiply: Area = 3 cm * 0.15 cm = 0.45 cm^2.

Example 11: Find the area, in square meters, of a rectangular wall that is 5 m high and 0.34 hm wide.

Solution:

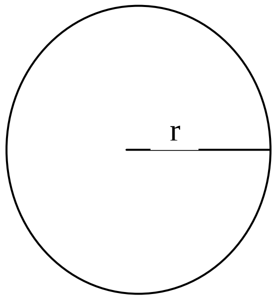
The width of the wall in meters is found by multiplying the width in hectometers times 100 (We are converting to a unit that is two powers of ten smaller). Thus the width is:

Width = 0.34 * 100 = 34 m.

Then find the area by multiplying length times width.

Area = 5 m * 34 m = 170 m^2.

The area of a circle can also be found by the formula $A = \pi r^2$, where r is the radius of the circle and π is approximately 3..14 (or use the π key on your calculator).



Example 12: Find the area, in square kilometers, of a circular region with a radius of 4 km.

Solution:

Evaluate the formula $A = \pi r^2$ when r = 4.
$A = \pi\left(4\right)^2 = 16\pi = 50.26$ rounded to 2 decimal places.

The area of the region is 50.26 km^2.

Volume

The metric unit for volume is the liter. However, volume is often calculated from the dimensions of a solid in cubic length units. The volume of 1 cubic meter is the volume of a cube with sides of length 1 meter. The volume of 1 cubic decimeter is the volume of a cube with sides of length 1 decimeter. The liter is equal to 1 cubic decimeter. The general convention is that volume of liquids is expressed in liters and volume of solids is expressed in cubic length units.

Example 13: The volume of a cube with sides of length 1 centimeter is one milliliter. Why?

Solution:

Since 1 centimeters is equal to $\frac{1}{10}$ decimeter, a cube with sides of length 1 cm has volume

$$\frac{1}{10}dm \times \frac{1}{10}dm \times \frac{1}{10}dm = \frac{1}{1000}dm^3 = \frac{1}{1000}l$$

The unit that is one-thousandth of a liter is a milliliter. Therefore, a cubic centimeter is one milliliter.

You will need to be able to calculate the volume of a rectangular box and a cylinder using the basic geometry formulas for volume and their dimensions.

Example 14: Find the volume, in cubic meters, of a rectangular swimming pool that is 10 m wide, 20 m long and 2 m deep.

Solution:

The volume of a rectangular box is length times width times height (depth). So,

$V = 10 \times 20 \times 2 = 400 \ m^3$.

Example 15: Find the volume of a cylindrical soda can with radius 5 cm and height 20 cm.

Solution:

The formula for the volume of a cylinder with height h and radius r is $V = \pi r^2 h$.
In this case, r = 5 and h = 20 and the volume will be in square centimeters.

$V = \pi (5)^2 (20) = 1571$.

The volume of the can is 1571 cm^3

If the dimensions are not all given in the same units, convert all measurements to the same unit, then use the formula to find the volume.

Example 16: Find the volume, in cubic centimeters, of a box that is 4.5 cm wide, 1.2 dm long and 0.3 m high.

Solution:

The dimensions of the box are not all given in centimeters, so convert all measurements to centimeters before using the volume formula.

Width = 4.5 cm is already in the appropriate units.
Length = 1.2 dm. To convert from decimeters to centimeters (moving to one unit smaller) multiply by 10. Thus, the length of the box is 1.2 * 10 = 12 cm.
Height = 0.3 m. To convert from meters to centimeters (moving to two units smaller) multiply by 100. Thus, the height of the box is 0.3 * 100 = 30 cm.

Volume = 4.5 cm * 12 cm * 30 cm = 1620 cm^3.