2.5 Order – Comparing Real Numbers

The Real number line is ordered: the smaller numbers are on the left and the larger numbers are on the right.

This fact allows us to insert an inequality symbol between two numbers that are not equal. There are four inequality symbols:

$A > B$	A is greater than B.
$A \geq B$	A is greater than B or A is equal to B.*
$A < B$	A is less than B.
$A \leq B$	A is less than B or A is equal to B.*

*These are generally shortened to

- A is "greater than or equal to" B ($A \geq B$) or
- A is "less than or equal to" B. ($A \leq B$)

Note that only one phrase need be true for the statement to be true in a logical sense.

It is true to write $5 \leq 7$ because 5 is less than 7. The real use of the two inequality symbols is for number line rays. If you want to discuss all the numbers that are less than 7 and to include 7 in the discussion, you want $\{x \mid x \leq 7\}$, this is much more efficient to say in set builder notation than to try to make a list or write it out in English.

Since we are working with numbers and NOT rays, we will use < and > without the additional equality statement.

How do you tell whether a number A is less than, more than, or equal to another number B? Well, you compare them by subtracting them in order – the number on the left minus the number on the right. You may also divide them: $\frac{A}{B}$.

Checking by subtraction:

If A – B is a positive number, then A is larger than B.
If A – B is 0, then A = B.
If A – B is a negative number, then A is less than B.

Checking by division (ONLY if A and B are both positive):

If $\frac{A}{B}$ is greater than one, then A is larger than B.

If $\frac{A}{B}$ is equal to 1, then A = B.

If $\frac{A}{B}$ is less than 1, then A < B.

Examples:

Insert the correct symbol: >, <, or =.

A.	5 ____ 3	5 – 3 = 2. Use >
B.	2^{-2} ___ 3^{-1}	$\frac{1}{4}-\frac{1}{3}=\frac{3}{12}-\frac{4}{12}=-\frac{1}{12}$. Use <
C.	-5^0 ___ 2	–1 – 2 = –3. Use <.
D.	$-\frac{1}{3}$ ___ $-\frac{1}{5}$	$-\frac{5}{15}--\frac{3}{15}=-\frac{5}{15}+\frac{3}{15}=-\frac{2}{15}$. Use <.
E.	.25 ___ 2.5×10^{-2}	$\frac{2.5(10^{-1})}{2.5(10^{-2})}=10$. Use >.

If you have irrational numbers, remember the earlier exercise in the first section – the square roots are also ordered smallest to largest and can be converted easily to a mixed number*

Examples:

Insert the correct symbol: >, <, or =.

A.	$\sqrt{23}$ ____ $\sqrt{15}$	Use >.
B.	$19^{\frac{1}{2}}$ _____ $16^{\frac{1}{2}}$	Use >

C. $2\sqrt{5}$ ______ $3\sqrt{5}$ $2\sqrt{5}-3\sqrt{5}=-\sqrt{5}$. Use <.

D. $\sqrt{12}$ _____ 3 $\sqrt{12}\approx 3\frac{3}{7}$*. Use >.

E. $\sqrt{7}$ ___ $\frac{14}{5}$ $\approx \frac{13}{5}^{*} \div \frac{14}{5} = \frac{13}{5}\cdot\frac{5}{14} = \frac{13}{14}$. Use <.

* A quick review on converting a square root to a nearby rational number:

Take the number under the radical and put the nearest smaller perfect square on the left and the nearest larger perfect square on the right, putting in a period for each natural number between them:

9 . . 12 . . . 16 Note that the square root of the perfect square on the left is the leading natural number in the mixed number. In this case: 3.

Count the number of steps from the leftmost number to the rightmost. Here: 7. This is the denominator of the fraction.

Count the number of steps from the leftmost number to the radicand, the number under the radical. Here: 3. This is the numerator of the fraction.