Section 2.4

Scientific Notation

Scientific Notation is used often in engineering and the physical sciences where very large and very small numbers occur naturally.

Distance to the sun

Size of a cell wall

We may rewrite the number in the standard scientific notation form:

$$a \times 10^n$$

with n, an integer and $1 \le |a| < 10$ and a is a decimal.

Two examples of small numbers written in scientific notation are

$$\frac{3}{4} = 7.5 \times 10^{-1}$$

$$-100 = -1.0 \times 10^2$$

Here are the steps to rewrite a number in decimal form to scientific notation:

Steps:

1. Put a tic mark to the right of the first non-zero digit in the number.
   Put in a decimal point on the right of the last digit if there is no decimal point showing; call this the original decimal point, else use the decimal point that is in the original number for the next step.l
2. Count the number of digits from the tic mark to the original decimal point.
   Use this number with the proper sign (+/ –) as the exponent on the 10, the "n" in the exponent place.
3. Drop the original decimal point and replace the tic mark with a new decimal.
   This number is now "a".
4. Put an " $\times$" in between the number that is "a" and the 10^n

REMEMBER that decimal numbers are arranged in descending place value order left to right...counting left to the decimal means you are working with a small number (a negative exponent) and counting right to the decimal means you are working with a big number (a positive exponent).

Also note that sometimes the number is given in a form other than a decimal. Then there is a 0^{th} step: changing the number to a decimal.

Examples:

.001 is a small number and .001 = $\frac{1}{1000} = 1.0 \times 10^{-3}$

1. .001'
2. There are 3 places to the left from the tic mark to the original decimal.
   Use an exponent of –3.
3. 1.0
4. 1.0×10^{-3}

10,000 is big number and $10,000.0 = 1.0 \times 10^{4}$

1. 1'0,000.
2. There are 4 places to the right from the tic mark to the original decimal.
   Use an exponent of 4.
3. 1.0
4. 1.0×10^{4}

$\frac{3}{4}$

0. .75
1. .7'5
2. There is one place from the tic mark to the left to the original decimal point.
   Use an exponent of –1 as "n".
3. 7.5
4. 7.5×10^{-1}

100

1. 1'00.
2. There are 2 places from the tic mark to the right to the decimal point we had to add.
   Use an exponent of 2 as "n".
3. 1.00
4. 1.0×10^{2}

580,000,000

1. 5’80,000,000.
2. 8 places to the right, n = 8
3. 5.8
4. 5.8×10^{8}

$-\frac{3}{5}$

0. $-\frac{3}{5} = -.6$
1. –.6’
2. 1 place to the left, n = –1
3. – 6.0
4. -6.0×10^{-1}

$2(5)^{-1}$

0. $2(5)^{-1} = \frac{2}{5} = .4$
1. .4’
2. 1 place to the left, n = – 1
3. 4.0
4. 4.0×10^{-1}

5

1. 5’.
2. no places to the left or right, n = 0
3. 5.0
4. 5.0×10^{0}

Sometimes there are quite a few digits in a number. We will follow a convention of rounding to the second place after the scientific notation decimal point to rewrite number with many different digits.

The Rounding convention will be: check the digit in the 3rd place after the scientific notation decimal point – if it is 5 or larger round the digit in the 2nd place up one, else do not change the 2nd digit.

Examples:

.00098567183

1. .0009’8567183
2. 4 places to the left, n = –4
3. 9.86567183 becomes 9.86 with rounding up
4. 9.86×10^{-4}

10,339,756	1.	1’0,339,756
	2.	7 places to the right, n= 7
	3.	1.0339756 becomes 10.03 with no rounding up
	4.	1.03×10^{7}
-2^{-5}	0.	$-2^{-5} = -\frac{1}{32} = -.03125$
	1.	–.03’125
	2.	2 places to the left, n = – 2
	3.	– 3.125 becomes –3.13
	4.	-3.13×10^{-2}
$\frac{1}{3}$	0.	$\frac{1}{3} = .33\overline{3}$
	1.	$.3’33\overline{3}$
	2.	1 place to the left, n = –1
	3.	$3.33\overline{3} \approx 3.33$
	4.	3.33×10^{-1}

Problem solving with very large and very small numbers:

We will make extensive use of the Rules for Exponents in calculating the answers to questions with very small and very large exponents. If you round your answer, do so in the very last step and replace the “=” with a “$\approx$”.

Example with no rounding:

Simplify the following expressions.

A. $\dfrac{9,000,000(.000042)}{.003(20,000,000)}$

first change all the numbers to scientific notation

$$\frac{9,000,000(.000042)}{.003(20,000,000)} = \frac{9.0\times10^{6}(4.2\times10^{-5})}{3.0\times10^{-2}(2.0\times10^{9})}$$

Then group the numbers so that digits are in the front of the numbers and base 10 numbers are listed last. Suppress the ".0" and the "×" in each expression.

$$\frac{9(4.2)(10^{6})(10^{-5})}{3(2)(10^{-2})(10^{9})}$$

Now, cancel what you can from the front and combine all the exponents into one base 10 number.

$3(2.1)(10^{(6-5)-(-2+9)})$ Now finish up on the calculation.

6.3×10^{-6}

B. Working with numbers in decimal form can be made easier by converting to "almost scientific notation". Note that we change each number in the problem to an integer and use modified version of scientific notation to work this problem easily. The powers of 10 are just those that work with an integer and not those that make "a", the leading number in actual scientific notation. Be sure to report your answer in real scientific notation, though.

$$\frac{(.35)^{2}(9,000)}{(.7)(150)}=\frac{(35\times10^{-2})^{2}(9\times10^{3})}{(7\times10^{-1})(15\times10)}=$$

$$\frac{(5\cdot7)^{2}(3)^{2}(10^{-4+3})}{7(3)(5)(10^{0})}=\frac{5^{2}(7^{2})(3^{2})(10^{-1})}{7(3)(5)}=$$

$$5(7)(3)(10^{-1})=105(10^{-1})=1.05(10^{2})(10^{-1})=$$

1.05×10

C. Sometimes, numbers have to be converted to a decimal to begin working the problem.

$$\frac{5^{-1}(14,000)}{.0007}=\frac{.2(14)(10^{3})}{7(10^{-4})}=\frac{2(14)10^{-1+3}}{7(10^{-4})}=$$

$2(2)(10^{2--4})=4.0\times10^{6}$

Example with rounding:

$$\frac{.0000008416(522,007)}{2{,}173{,}000(.0006)} =$$

$$\frac{8.416\times10^{-7}(5.22007\times10^{5})}{2.173\times10^{6}(6\times10^{-4})} =$$

$$\frac{8.416(5.22007)(10^{-7})(10^{5})}{2.173(6)(10^{6})(10^{-4})} =$$

$$\frac{8.416(5.22007)}{2.173(6)}\times10^{(-7+5)-(6-4)} =$$

$$3.369543574\times10^{-4} \approx 3.37\times10^{-4}$$

Of course, you may use a calculator on the numbers in this type of problem in the homework. This will be noted in the instructions.