Section 3.1

Variables, Expressions and the Order of Operations Rule

In the next several chapters, we'll cover some topics that you usually think of when you hear the word algebra: solving equations and inequalities, functions, graphing functions and linear inequalities and solving systems of equations and inequalities.

Generally speaking, algebra is concerned with the relationships between numbers and quantities and with a structure needed to define those relationships. Much like learning a language, learning algebra requires knowing some vocabulary, symbols and rules and being able to use them successfully.

The word "algebra" comes from the Arabic word *al-jabr* which was part of a ninth century book title on calculation using "transposition and reduction."

What is the benefit of studying algebra? Using algebra, you'll be able to solve a variety of problems systematically and efficiently

Variables, Expressions and the Order of Operations Rule

Arithmetic concerns itself with numbers and the basic operations, addition subtraction, multiplication and division, that are used to combine them. Sometimes we need a more generalized statement.

Suppose we know that Ben has $50 more that Dianne does. This is nice information, but unless we know how much money Dianne has, we can't state how much Ben has.

For example, if Dianne has $30, then Ben has $30 + $50, or $80.
If Dianne has $300, then Ben has $300 + 30, or $330.

To state this more generally we can express the amount of money Ben has using a variable: $x + 30$, where x represents the amount of money Dianne has. Once we know x, we can find the amount of money Ben has.

We'll use variables throughout this part of the course:

Definition: A **variable** is a symbol, usually a letter of the alphabet, that is used to replace a number.

Typically, we'll use x and y to represent variables, but we can use any letter of the alphabet as a variable. We can choose letters that are appropriate to a problem we're solving. For example, in a problem involving the perimeter of a rectangle, we'd likely use the formula, $P = 2l + 2w$, where P represents the perimeter, and l and w represent the length and width of the rectangle, respectively.

When we combine numbers, variables and operation symbols together, we can write **expressions** (or **algebraic expressions)**. Here are some examples of expressions:

$$4+5\cdot 7$$

$$2x-5$$

$$ab+3cd$$

$$\frac{x-3y}{ab+2}$$

One of the first things you need to be able to do accurately and efficiently is to evaluate expressions. When we want to evaluate an expression such as $4+5\cdot 7$, there are two ways we might approach the problem, but only one of them is correct.

So, do we add 4 and 5 and then multiply by 7? If we do this, we'll get 63.

Or, do we multiply 5 by 7 and then add 4? If we do this, we'll get 39.

The problem can't have two answers. Each algebraic expression must have a unique answer. To make sure that this will always be the case, we agree upon an order in which to perform the operations.

1. First, perform any multiplications and divisions in the order in which they appear from left to right.
2. Then, perform any additions and subtractions in the order in which they appear from left to right.

So then, the correct answer in our example above is 39: we should multiply first and then add 4.

Example 1: Evaluate: $15\div 5\cdot 2-3\cdot 6$

Solution:

$$\begin{aligned}15\div 5\cdot 2-3\cdot 6 &= 6-18\\ &= -12\end{aligned}$$

Sometimes, however, these two simple rules are not enough to be able to evaluate an expression. Sometimes expressions can contain exponents, and they require special treatment.

If we write 4^3, we mean $4 \cdot 4 \cdot 4$. In the expression 4^3, we call 4 the "base" and 3 the "power" or the "exponent." We would read this as "4 raised to the third power."

More generally if we have x^n, we mean *x* used as a factor *n* times. We call *x* the base and *n* the power or the exponent.

Example 2: Evaluate each:

a. -6^2

b. $(-6x)^0$

c. $-6x^0$

Solutions:

a. $-6^2 = -1 \cdot 6^2 = -1 \cdot 36 = -36$

b. $(-6x)^0 = 1$

c. $-6x^0 = -6 \cdot x^0 = -6 \cdot 1 = -6$

Sometimes problems contain grouping symbols. These symbols serve to change the order in which operations should be performed.

In an earlier example, we saw that $4 + 5 \cdot 7 = 39$, since we should multiply first and then add. If we needed to work this problem in another order, we would use parentheses to modify the order that was listed above. Parentheses or other grouping symbols indicate that the operation(s) inside the parentheses should be done first.

If we write $(4+5) \cdot 7$, then we should add first, then multiply by 7. So $(4+5) \cdot 7 = 63$.

The Order of Operations Rule

We can summarize the rules in this section as follows:

1. Perform any operations that are inside parentheses or other grouping symbols (using the rest of the rules listed below, if needed).
2. Perform any exponential operations given in the problem (i.e., raise numbers to powers).
3. Multiply and divide in the order in which these operations appear from left to right.
4. Add and subtract in the order in which these operations appear from left to right.

An easy way to remember the Order of Operations Rule is by using the acronym ***PEMDAS*** (***P***arentheses, ***E***xponents, ***M***ultiplication and ***D***ivision, ***A***ddition and ***S***ubtraction).

Example 3: Evaluate: $5^2 - 3(8-3) + 9$

Solution:

$$\begin{aligned} 5^2 - 3(8-3) + 9 &= 5^2 - 3(5) + 9 \\ &= 25 - 3(5) + 9 \\ &= 25 - 15 + 9 \\ &= 19 \end{aligned}$$

Example 4: Evaluate: $\dfrac{2^3 - 3\cdot 2}{(1+5)\div 2}$

Solution:

$$\begin{aligned} \frac{2^3 - 3\cdot 2}{(1+5)\div 2} &= \frac{2^3 - 3\cdot 2}{6\div 2} \\ &= \frac{8 - 3\cdot 2}{6\div 2} \\ &= \frac{8-6}{3} \\ &= \frac{2}{3} \end{aligned}$$

Example 5: Evaluate: $25 - 3(2 - 3^3) + (5-1)\div 8\cdot 10$

Solution:

$$\begin{aligned} 25 - 3(2 - 3^3) + (5-1)\div 8\cdot 10 &= 25 - 3(2-27) + 4\div 8\cdot 10 \\ &= 25 - 3(-25) + 4\div 8\cdot 10 \\ &= 25 + 75 + 5 \\ &= 105 \end{aligned}$$

Note: Multiplication and division are performed on the same step. You do NOT multiply in one step and then divide in another step.

So $10 \div 2 \cdot 5 = 25$, since $10 \div 2 = 5$ and then $5 \cdot 5 = 25$

Using the Order of Operations Rule

We can use the order of operations rule to evaluate expressions that are given in variables with appropriate values for the variables stated.

Example 6: Evaluate $2x^2 - 3xy + 5y^2$ if $x = -2$ and $y = -3$.

Solution:

First, substitute -2 for *x* and -3 for *y*:

$$2(-2)^2 - 3(-2)(-3) + 5(-3)^2$$

Now use the Order of Operations Rule to evaluate:

$$\begin{aligned} 2(-2)^2 - 3(-2)(-3) + 5(-3)^2 &= 2(4) - 3(-2)(-3) + 5(9) \\ &= 8 - 18 + 45 \\ &= 35 \end{aligned}$$

Example 7: Evaluate $\dfrac{2(ab - bc) + a}{b^2 - c^2}$ if $a = -5, b = 3$ and $c = -1$

Solution:

First, substitute -5 for *a*, 3 for *b* and -1 for *c*:

$$\begin{aligned} \frac{2\left(-5 \cdot 3 - 3(-1)\right) + (-5)}{3^2 - (-1)^2} &= \frac{2(-15 + 3) + (-5)}{3^2 - \left(-1\right)^2} \\ &= \frac{2\left(-12\right) + (-5)}{3^2 - \left(-1\right)^2} \\ &= \frac{2(-12) + (-5)}{9 - 1} \\ &= \frac{-24 + (-5)}{9 - 1} \\ &= \frac{-29}{8} \end{aligned}$$

We can also use substitution and the Order of Operations Rule to solve some word problems:

Example 8: A national cable company offers a promotion for its monthly digital cable television service for $49.95 per month. This includes two pay-per-view movies. Additional pay-per-view movies cost $4.95 each. What will be the pre-tax bill if you watch a total of 15 pay-per-view movies this month?

Define a variable, write an expression and use it to answer the question.

Solution:

Start by defining a variable. In this problem, you might define your variable as

let x = the number of movies I have to pay for

Then you can express the total bill as

$$4.95x + 49.95$$

In this problem, you watched a total of 15 pay-per-view movies, but two of them were included in your monthly service, so $x = 13$. You can substitute and evaluate:

$$4.95(13) + 49.95 = 64.35 + 49.95 = 114.30$$

So your pre-tax bill will be $114.30.