Section 3.2

Equations and Inequalities

So far, we have looked at algebraic expressions. Here are some examples

$$x+3$$
$$2ab-3c^2$$

If we compare these to language, we'll see that we can compare these to phrases. They aren't sentences because they lack verbs. In this section, we'll take our expressions and see what we can do to turn them into mathematical sentences.

Equations

We can join two algebraic expressions together with an equal sign if the quantity on the left side of the equal sign and the quantity on the right side of the equal sign represent the same number. We call such a sentence an **equation**.

Here are some examples of equations:

$$3+5=8$$
$$7y+3=34$$
$$x=5a+3b$$

It is easy for us to determine that the first sentence is always true; it is simply an elementary arithmetic fact. However, we do not have enough information to determine if the other two equations listed above are true or false. There are many values of *y* for which the second equation is not true, and only one value of *y* for which it is true. Similarly, whether or not the third equation is true depends on the values of *x, a,* and *b*.

If an equation contains a variable (or variables), and a value (or values) is suggested for the variable, we can determine if the value makes the equation true. If it does, we call it a **solution of the equation**.

Example 1: Determine if $x=3$ is a solution of the equation $x^2-4x=-3$.

Solution:

Substitute 3 for *x* in the equation:

$$3^2-4\cdot 3\overset{?}{=}-3$$
$$9-4\cdot 3\overset{?}{=}-3$$
$$9-12\overset{?}{=}-3$$
$$-3=-3$$

Since -3 = -3, we conclude that $x=3$ is a solution of the equation.

Example 2: Determine if $x = 3,\ y = -1$ is a solution of the equation $x^3 + 4xy - 2y^2 = 10$.

Solution:

Substitute 3 for *x* and -1 for *y* in the equation.

$$(3)^3 + 4(3)(-1) - 2(-1)^2 \stackrel{?}{=} 10$$
$$27 + 4(3)(-1) - 2(-1) \stackrel{?}{=} 10$$
$$27 - 12 + 2 \stackrel{?}{=} 10$$
$$17 \neq 10$$

Since 17 is not equal to 10, we can conclude that $x = 3,\ y = -1$ is not a solution of the equation.

Inequalities

There are other types of mathematical sentences. We use inequality symbols to describe the relationship between two quantities when they are not equal. We'll look at three different inequality symbols in this section:

$<$ means "is less than" so the quantity on the left is smaller than the quantity on the right

$>$ means "is greater than," so the quantity on the left is larger than the quantity on the right

$\neq$ means "is not equal to," so we know that the quantity on the left and the quantity on the right are not equal, but we don't know which one is larger

Here's a way to remember the meaning of the first two inequality symbols: Think of the symbol as being an arrow head. Then remember, the arrow always points to the ***smaller*** number.

Let's consider the inequality $-3 < 5 - 6$.

We can simplify the right hand side, so we'll have $-3 < -1$. The arrow is pointing towards -3, and -3 is smaller than -1, so this inequality is true.

Example 3: Determine if each statement is true or false.

a. $5 > 2 \cdot 7 \cdot 0$
b. $6 \cdot 4 \neq 3 \cdot 8$
c. $7 + 2(-4) < 5(3) - 4^2$

Solution:

a. We'll work out the right side of the inequality, then determine if the resulting statement is true.

$$5 > 2 \cdot 7 \cdot 0$$
$$5 > 14 \cdot 0$$
$$5 > 0$$

It is true that 5 is greater than 0. (Note that the arrow points to the smaller number.) So this statement is true.

b. We'll work out both the left side and the right side of the inequality, then determine if the resulting statement is true.

$$6 \cdot 4 \neq 3 \cdot 8$$
$$24 \neq 24$$

But 24 ***is*** equal to 24, so this statement is false.

c. We'll work out both the left side and the right side of the inequality, then determine if the resulting statement is true.

$$7 + 2(-4) < 5(3) - 4^2$$
$$7 + 2(-4) < 5(3) - 16$$
$$7 + -8 < 15 - 16$$
$$-1 < -1$$

This statement is not true, because -1 = -1.

If an inequality contains a variable (or variables), and a value (or values) is suggested for the variable, we can determine if the value(s) makes the equation true. If it does, we call it a **solution of the inequality.**

Example 4: Determine if $x = -1,\ y = 6$ is a solution of the inequality $5x + 3y > 3$.

Solution:

Substitute -1 for *x* and 6 for *y*. The evaluate the left side of the inequality. Finally compare the two numbers to determine if the inequality is true.

$$\begin{aligned} 5(-1) + 3(6) &> 3 \\ -5 + 18 &> 3 \\ 13 &> 3 \end{aligned}$$

Since 13 is, in fact, larger than 3, we can conclude that $x = -1,\ y = 6$ is a solution of the inequality.