Section 3.3

Solving Equations

Now we're ready to learn some techniques for solving equations that contain variables. Rather than just guessing at the value or values for *x* that will be solutions to an equation, we can use a set of rules for solving an equation. Our objective in solving an equation will be to isolate the variable. Once we've done that, we can determine the solution to the equation by simplifying the other side of the equation.

First some preliminaries.

Algebraic expressions are often made up of terms. For example, in the expression $5x+12y$, there are two terms, $5x$ and $12y$. A **term** is a number or a variable or the product (or quotient) of numbers and variables. Terms are connected in an expression by addition and/or subtraction signs. When a variable is multiplied by a number, the number is called a **coefficient**. A term that contains no variables is called a **constant.**

Using the Distributive Property

We saw earlier that we can distribute multiplication over addition. The distributive property of multiplication over addition states that, for real numbers *a*, *b*, and *c*:

$$a(b+c)=ab+ac$$

We can use this property to simplify algebraic expressions.

Example 1: Write without using parentheses: $7(2x+3)$

Solution: We need to apply the distributive property:

$$\begin{aligned} 7(2x+3) &= 7\cdot 2x+7\cdot 3 \\ &= 14x+21 \end{aligned}$$

Combining Like Terms

Two terms are considered to be **like terms** if they contain the same variables raised to the same powers. Here are some examples:

$4x$ and $9x$ are like terms: they both contain the variable *x* and in both cases, *x* is raised to the first power.

$16x^2$ and $12x^3$ are not like terms: while they both contain the same variable (*x*) the powers are not the same.

Example 2: Determine if each pair of terms are like terms.

a. $4xy$ and $11yx$
b. $3x^2y^3$ and $-5x^3y^2$
c. $10a$ and $14b$

Solution:

a. $4xy$ and $11yx$ are like terms. Both contain the variables *x* and *y* and each variable is raised to the first power. Note that the order in which the variables are written does not matter.

b. $3x^2y^3$ and $-5x^3y^2$ are not like terms. While they both contain the variables *x* and *y*, in the first term *x* is raised to the second power and *y* is raised to the third power and in the second term, *x* is raised to the third power and *y* is raised to the second power.

c. $10a$ and $14b$ are not like terms. They do not contain the same variables.

We will need to **combine like term**s. To do this we can use the distributive property in reverse, $ab+ac=(b+c)a$.

Example 3: If possible, combine like terms: $6x+18x$

Solution: $6x$ and $18x$ are like terms, so we can combine them together:

$$\begin{aligned} 6x+18x &= (6+18)x \\ &= 24x \end{aligned}$$

Note that when you combine the terms together, the variables do not change; only the coefficient changes.

Example 4: If possible, combine like terms: $7x+4y-9+2x-6y$

Solution: $7x$ and $2x$ are like terms, as are $4y$ and $-6y$, so we can combine them together. Note that we cannot combine the constant, 9, with any other terms.

$$\begin{aligned} 7x+4y-9+2x-6y &= 7x+2x+4y-6y+9 \\ &= (7+2)x+(4-6)y+9 \\ &= 9x-2y+9 \end{aligned}$$

Example 5: Write without using parentheses and combine like terms, if possible:

$$4-2(6x-4y)+3(x-5y)$$

Solution: We need to apply the distributive property twice.

$$4-2(6x-4y)+3(x-5y)=4-12x+8y+3x-15y$$

Now we can rearrange the terms and then combine like terms:

$$\begin{aligned} 4-2(6x-4y)+3(x-5y) &= 4-12x+8y+3x-15y \\ &= -12x+3x+8y-15y+4 \\ &= (-12+3)x+(8-15)y+4 \\ &= -9x-7y+4 \end{aligned}$$

Solving Equations

We can use any of these properties to help us solve equations:

If x is a variable and a, b, and c are real numbers, with $a \neq 0$. Then,

if $x+b=c$, then $x+b-b=c-b$ and $x=c-b$.
if $x-b=c$, then $x-b+b=c+b$ and $x=c+b$.
if $ax=b$, then $\frac{1}{a}\cdot ax=\frac{1}{a}\cdot b$ and $x=\frac{b}{a}$.
if $\frac{x}{a}=b$, then $a\cdot\frac{x}{a}=a\cdot b$ and $x=ab$

Example 6: Solve each equation:

a. $x-12=-23$
b. $5+x=-5$
c. $-4x=20$
d. $\frac{x}{5}=-2$

Solution:

a. To solve $x-12=-23$, we'll need to add 12 to both sides of the equation:

$$\begin{aligned} x-12 &= -23 \\ x-12+12 &= -23+12 \\ x+0 &= -11 \\ x &= -11 \end{aligned}$$

b. To solve $5+x=-5$, we'll need to subtract 5 from both sides of the equation:

$$\begin{aligned} 5+x &= -5 \\ 5+x-5 &= -5-5 \\ x+0 &= -10 \\ x &= -10 \end{aligned}$$

c. To solve $-4x = 20$, we'll need to multiply both sides of the equation by $\frac{-1}{4}$. We can also think of this as dividing both sides of the equation by -4.

$$\begin{aligned} -4x &= 20 \\ \frac{-1}{4}\cdot -4x &= \frac{-1}{4}\cdot 20 \\ 1x &= -5 \\ x &= -5 \end{aligned}$$

d. To solve $\frac{x}{5} = -2$, we'll need to multiply both sides of the equation by 5.

$$\begin{aligned} \frac{x}{5} &= -2 \\ 5\cdot\frac{x}{5} &= 5\cdot(-2) \\ 1x &= -10 \\ x &= -10 \end{aligned}$$

Most often we will use these properties in combination.

Example 7: Solve for *x*: $-5x+3=-17$

Solution: First, we'll subtract 3 from both sides of the equation. Then we'll multiply both sides of the resulting equation by $\frac{-1}{5}$.

$$
\begin{aligned}
-5x+3&=-17\\
-5x+3-3&=-17-3\\
-5x+0&=-20\\
-5x&=-20\\
\frac{-1}{5}\cdot -5x&=\frac{-1}{5}\cdot(-20)\\
1x&=\frac{20}{5}\\
x&=4
\end{aligned}
$$

Note, you can check your solution by substituting your answer into the original equation and evaluating. So for example 7,

$$
\begin{aligned}
-5(4)+3&\stackrel{?}{=}-17\\
-20+3&\stackrel{?}{=}-17\\
-17&=-17
\end{aligned}
$$

We can solve some harder equations. In the next example, we'll be need to distribute and combine like terms.

Example 8: Solve for *x*: $5x+3(4-2x)=9$

Solution: First we'll need to distribute:

$$
\begin{aligned}
5x+3(4-2x)&=9\\
5x+12-6x&=9
\end{aligned}
$$

Then we can combine like terms:

$$
\begin{aligned}
5x-6x+12&=9\\
-x+12&=9
\end{aligned}
$$

Then we can solve the resulting two-step equation:

$$
\begin{aligned}
-x+12-12&=9-12\\
-x+0&=-3\\
-x&=-3\\
(-1)(-x)&=(-1)(-3)\\
1x&=3\\
x&=3
\end{aligned}
$$

We can check the result:

$$\begin{aligned} 5(3)+3(4-2(3)) &\stackrel{?}{=} 9 \\ 15+3(4-6) &\stackrel{?}{=} 9 \\ 15+3(-2) &\stackrel{?}{=} 9 \\ 15-6 &\stackrel{?}{=} 9 \\ 9 &= 9 \end{aligned}$$

In some equations, there will be variables present on both sides of the equal sign. Your initial task will be to get all of the terms containing variables on the same side of the equal sign. Then you can use methods you've already seen to finish the problem.

Example 9: Solve for *x*: $5x+6=-3x+12$

Solution: First we'll get both terms containing *x* onto the left side of the equation. Then we can solve the resulting two-step equation.

$$\begin{aligned} 5x+6 &= -3x+12 \\ 5x+3x+6 &= -3x+3x+12 \\ 8x+6 &= 0+12 \\ 8x+6 &= 12 \\ 8x+6-6 &= 12-6 \\ 8x+0 &= 6 \\ 8x &= 6 \\ \frac{1}{8}\cdot 8x &= \frac{1}{8}\cdot 6 \\ 1x &= \frac{6}{8} \\ x &= \frac{3}{4} \end{aligned}$$

Sometimes problems will contain fractions or decimals. It is often easiest to clear the problem of fractions or decimals and then work with the resulting equation.

Example 10: Solve for *x*: $\frac{1}{3}(x+2)-\frac{1}{2}(4x+3)=\frac{3}{4}(x-5)$

Solution: We'll need to find the least common denominator for the problem before we can start solving it. Our denominators are 3, 2 and 4. We need the smallest number that all three of them can divide into without a remainder. That number is 12. So the LCD is 12.

We'll multiply both sides of the equation by 12.

$$12\left[\frac{1}{3}(x+2)-\frac{1}{2}(4x+3)\right]=12\left[\frac{3}{4}(x-5)\right]$$

Next, we can distribute and simplify:

$$12\left[\frac{1}{3}(x+2)-\frac{1}{2}(4x+3)\right]=12\left[\frac{3}{4}(x-5)\right]$$
$$\frac{12}{3}(x+2)-\frac{12}{2}(4x+3)=\frac{36}{4}(x-5)$$
$$4(x+2)-6(4x+3)=9(x-5)$$

Now, we can distribute and combine like terms:

$$4x+8-24x-18=9x-45$$
$$-20x-10=9x-45$$

Next, we'll need to isolate the variable:

$$-20x-10=9x-45$$
$$-20x-9x-10=9x-9x-45$$
$$-29x-10=-45$$
$$-29x-10+10=-45+10$$
$$-29x+0=-45+10$$
$$-29x=-35$$
$$\frac{1}{-29}\cdot -29x=\frac{1}{-29}\cdot -35$$
$$1x=\frac{35}{29}$$
$$x=\frac{35}{29}$$

A calculator will come in quite handy on the next example.

Example 11: Solve for x: $0.25(x-1.3)+0.6(2.4x-3.6)=0.78(5-1.2x)$

Solution: We can start by multiplying by 100. This will eliminate all of the decimal numbers that are outside the parentheses:

$$100[0.25(x-1.3)+0.6(2.4x-3.6)]=100[0.78(5-1.2x)]$$
$$25(x-1.3)+60(2.4x-3.6)=78(5-1.2x)$$

Next, we can distribute:

$$25(x-1.3)+60(2.4x-3.6)=78(5-1.2x)$$
$$25x-32.5+144x-216=390-93.6x$$

Now we can combine line terms:

$$25x-32.5+144x-216=390-93.6x$$
$$169x-248.5=390-93.6x$$

Next, we can add 93.6*x* to both sides of the equation:

$$169x-248.5=390-93.6x$$
$$169x-248.5+93.6x=390-93.6x+93.6x$$
$$262.6x-248.5=390$$

At this point, we have a two-step equation to solve:

$$262.6x-248.5=390$$
$$262.6x-248.5+248.5=390+248.5$$
$$262.6x=638.5$$
$$\frac{1}{262.6}\cdot 262.6x=\frac{1}{262.6}\cdot 638.5$$
$$x=\frac{638.5}{262.6}$$
$$x\approx 2.4315$$