Section 3.5

Using Formulas

You have probably used formulas since elementary school. You may have used formulas to find the area or perimeter of a rectangle and the circumference of a circle. You may have changed degrees Fahrenheit to degrees Celsius using a formula. Maybe you have computed interest using a formula.

In this section, we'll take a look at many different types of formulas. Some will be familiar, and some will not. Your objective will be to select the correct formula from a list that's provided, then choose the appropriate values from given information and finally use your calculator to evaluate the formula at the values given.

Using Formulas

A **formula** is a statement using mathematical symbols that gives the relationship between quantities.

For example, the distance around a rectangle (the perimeter) is always the sum of the lengths of the four sides. A rectangle has two pairs of sides that each have the same length. So we can generalize this relationship by the formula,

$$Perimeter = 2 * length + 2 * width$$

The use of variables makes the formula much easier to use:

$$P = 2l + 2w$$

Then, if we know the length and the width of a certain rectangle, we can easily compute its perimeter using this formula.

Example 1: If the length of a rectangle is 28 inches and the width of the rectangle is 17 inches, find the perimeter:

Solution:

Since this problem involves perimeter of a rectangle, we will need to use that formula.

$$\begin{aligned} P &= 2l + 2w \\ P &= 2 \cdot 28 + 2 \cdot 17 \\ P &= 56 + 34 \\ P &= 90 \end{aligned}$$

So we can conclude that the perimeter is 90 inches.

Formulas are often used to help solve real world problems.

Example 2: A rancher needs to fence in a grazing area for some cattle. He plans to fence in a rectangular region whose width is 1200 yards and length is 2000 yards. The fencing material he wants to use costs $15 per yard. How much will the fencing material cost?

Solution:

First, we'll need to determine how much fencing material is needed. Since this problem involves perimeter of a rectangle, we will need to use that formula.

$$P = 2l + 2w$$
$$P = 2 \cdot 2000 + 2 \cdot 1200$$
$$P = 4000 + 2400$$
$$P = 6400$$

We can conclude that the rancher needs to buy 6400 yards of fencing. Now we can compute cost.

$$Cost = 6400(15)$$
$$Cost = 96000$$

The cost to fence in this region will be $96,000.

Here is a list of formulas related to geometric shapes. Note that in the formulas, *P* refers to perimeter, *A* refers to area and *V* refers to volume. We call the distance around a circle its circumference, so *C* refers to this quantity.

Perimeter, Area and Volume Formulas

Square: $P = 4s$, $A = s^2$

Rectangle: $P = 2l + 2w$, $A = lw$

Parallelogram: $A = bh$

Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Triangle: $A = \frac{1}{2}bh$

Circle: $C = 2\pi r$, $A = \pi r^2$

Closed Box: $A = 2lw + 2lh + 2wh$, $V = lwh$

Sphere: $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

Cone: $V = \frac{1}{3}\pi r^2 h$

Cylinder: $A = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

Note that when finding area, you will give your answer in terms of square units (or $units^2$) and when finding volume, you will give your answers in terms of cubic units (or $units^3$). Area and circumference answer should be stated using the given units.

Example 3: Find the surface area and volume of a closed box with length 14 cm, width 8 cm and height 4 cm.

Solution:

To find the surface area, we'll need to use the formula $A = 2lw + 2lh + 2wh$. In the problem, we have

$$l = 14$$
$$w = 8$$
$$h = 4$$

We can substitute these values into the formula and then evaluate:

$$A = 2lw + 2lh + 2wh$$
$$A = 2(14)(8) + 2(14)(4) + 2(8)(4)$$
$$A = 224 + 112 + 64$$
$$A = 400$$

The surface area of the box is $400\ cm^2$.

To find the volume, we'll need to use the formula $A = lwh$. In the problem, we have

$$l = 14$$
$$w = 8$$
$$h = 4$$

We can substitute these values into the formula and then evaluate:

$$A = lwh$$
$$A = 14(8)(4)$$
$$A = 448$$

The volume of the box is $448\ cm^3$.

Example 4: Find the volume of a sphere with radius $\frac{5}{2}$ inches.

Solution:

To find the volume, we'll need to use the formula $V = \frac{4}{3}\pi r^3$.

In the problem, we are told that the radius of the sphere is $\frac{5}{2}$ inches.

We'll substitute this quantity into the formula and then evaluate:

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi\left(\frac{5}{2}\right)^3$$
$$V = \frac{4}{3}\pi\left(\frac{125}{8}\right)$$
$$V = \frac{500}{24}\pi$$
$$V = \frac{125}{6}\pi$$

The volume of the sphere is $\frac{125}{6}\pi$ cubic inches. You can convert this answer to a decimal approximation using your calculator: $\frac{125}{6}\pi$ cubic inches ≈ 65.450 cubic inches.

Miscellaneous Formulas

Exponential Growth and Decay

Growth: $P(t) = P_0 e^{kt}$

Decay: $P(t) = P_0 e^{-kt}$

P_o = initial quantity, k = growth/decay constant, t = time

Fahrenheit to Celsius and Celsius to Fahrenheit

Fahrenheit to Celsius: $C = \frac{5}{9}(F - 32)$

Celsius to Fahrenheit: $F = \frac{9}{5}C + 32$

Pythagorean Theorem

In right triangle *ABC* with $\angle C = 90°$, $a^2 + b^2 = c^2$.

Interest and Financial Formulas

In these formulas, *P = present value (or amount invested), A = accumulated amount,*

Simple interest: $I = Prt$
where I = interest earned, P = amount invested, r = annual interest rate, t = time in years

Future value (simple interest): $A = P(1 + rt)$
where variables are as defined above

Future value (comp. interest): $A = P(1+i)^n$, where $i = \frac{r}{m}$ and $n = mt$
where variables are as defined above, m = number of compounding periods per year

Present value (comp. interest): $P = A(1+i)^{-n}$, where $i = \frac{r}{m}$ and $n = mt$
where variables are as defined above

Future value (annuity): $S = R\left[\frac{(1+i)^n - 1}{i}\right]$, where $i = \frac{r}{m}$ and $n = mt$
where variables are as defined above, S = future value and R = amount of periodic payment

Present value (annuity): $P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$, where $i = \frac{r}{m}$ and $n = mt$
where variables are as defined above

Monthly payment (loan): $R = \frac{Pi}{1-(1+i)^{-n}}$, where $i = \frac{r}{m}$ and $n = mt$ where variables are as defined above

Example 5: The temperature in Houston on a hot summer afternoon was recorded as $99°F$. What was the temperature on the Celsius scale?

Solution:

We are given the temperature on the Fahrenheit scale and are asked to convert it to the Celsius scale. We'll need to use the formula $C = \frac{5}{9}(F - 32)$.

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}(99 - 32)$$
$$C = \frac{5}{9}(67)$$
$$C = \frac{335}{9} \approx 37.2$$

The temperature was about $37.2°C$.

Example 6: Suppose you borrowed $25,000 to buy a new car. If your interest rate was 5.9% compounded monthly for 6 years, what was your monthly car payment?

Solution:

We want to compute a monthly payment, so we'll need to use the formula $R = \frac{Pi}{1-(1+i)^{-n}}$, where $i = \frac{r}{m}$ and $n = mt$. In this problem, we are given

$$P = 25{,}000$$
$$r = .059$$
$$m = 12$$
$$t = 6$$

So $i = \frac{.059}{12}$ and $n = 12(6) = 72$.

We can substitute these values into the formula:

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$R = \frac{25000\left(\frac{.059}{12}\right)}{1-\left(1+\frac{.059}{12}\right)^{-72}}$$
$$R = 413.14$$

The monthly payment is $413.14.

Is this a reasonable answer? Recall that you will make 72 payments of $413.14. That means you will pay a total of $29,746.08 to repay the loan. This includes the repayment of the amount borrowed plus the interest on the loan. Since you borrowed, $25,000, the amount of interest paid was $4,746.08. This seems reasonable. If the sum of payments was less than the amount borrowed, you should be suspicious of your answer.

Example 7: Suppose you put $300 per month into a savings account that pays 4% annual interest compounded monthly. How much money will be in the account after 5 years, if you don't take any money out?

Solution:

This problem is asking us to find the future value of a savings account. We have three future value formulas, so we must first determine which of the formulas to use. The problem refers to compound interest, so we can eliminate the simple interest formula. Let's look at the other two formulas. The Future Value (compound interest) formula is used when there is one lump-sum deposit. That is not what we have in this problem. We have monthly payments of $300. So we can rule out that formula. The final formula is the Future Value (annuity) formula, which contemplates computing the future value when we're making periodic payments. This is the correct formula to use.

We are given

$$R = 300$$
$$r = 0.04$$
$$m = 12$$
$$t = 5$$

so we can compute

$$i = \frac{r}{m} = \frac{.04}{12}$$
$$n = mt = 5(12) = 60$$

Now we can substitute the values into the formula:

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$S = 300\left[\frac{\left(1+\frac{.04}{12}\right)^{60} - 1}{\left(\frac{.04}{12}\right)}\right]$$
$$S = 19889.69$$

After five years, there will be $19889.69 in the account.

Example 8: Population grows exponentially. Suppose the population of a city was 125,000 people in 2005. If the growth constant is 0.0625, what would you predict the population will be in 2012?

Solution:

This is an exponential growth problem, so we'll use the formula $P = P_o e^{kt}$. We are given the values

$$P_o = 125{,}000$$
$$k = 0.0625$$

and since our beginning reference point was in 2005, we can find *t* by subtracting 2012 – 2005. So $t = 7$.

Substitute these values into the formula and evaluate:

$$P = P_o e^{kt}$$
$$P = 125{,}000e^{0.0625(7)}$$
$$P \approx 193{,}604$$

We'll predict that the population of the city will be about 193,604 in 2012.