

## Section 3.6

# Solving Problems Using Equations

We can model a real world situation using an equation, and then we can solve the equation to help us solve the problem.

When given a real world problem to solve, our first task will usually be to write the equation that we need to use to be able to solve the problem. Once we have the equation written, we can solve it and then use the results to answer the question that was posed.

Writing the equation is often the more difficult part. Here are some tips to help you master this part of the process.

1. Read the problem carefully until you understand what you are being asked to find. You may need to read it several times.
2. If appropriate, draw a picture. If the problem is about the area of a rectangle, for example, you should draw and label a rectangle.
3. Define a variable. Write it down using words (example: let  $x$  = width of the rectangle).
4. Write the equation. Keep in mind that your equation may involve a familiar formula.
5. Solve the equation.
6. Use your solution to the equation to answer the question that was asked in the problem.

Keep in mind that the best way to improve your problem-solving skills is to get more practice solving these every day problems!

**Example 1:** Suppose your test grades in your psychology class are 84, 97, 93 and 89. You'd like to have a 92 test average after taking the fifth test. Is that possible? If so, what grade will you need to earn on the fifth test.

Solution:

We're trying to find the grade needed on test 5 in order to have a 92 test average.

We're asked if this is possible, but we'll defer answering that question until we find the needed grade on test 5. If that grade is 100 or less, then we know it's possible to get to the 92 average. If the grade is over 100, then the 92 average is not possible.

We can define a variable.

let  $x$  = grade on test 5

To average the test grades, we'll need to add them up and divide by 5. We want this quantity to equal 92.

$$\frac{84 + 97 + 93 + 89 + x}{5} = 92$$

Now we can solve the equation:

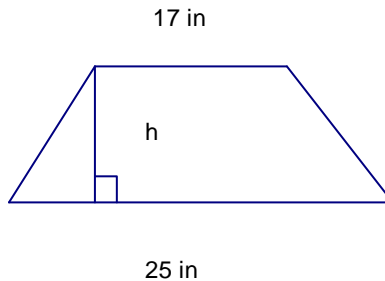
$$\begin{aligned}\frac{84 + 97 + 93 + 89 + x}{5} &= 92 \\ \frac{363 + x}{5} &= 92 \\ 363 + x &= 460 \\ x &= 97\end{aligned}$$

Now we can answer both questions. Yes, it will be possible to earn a 92 test average. You will need to score 97% on test 5 in order to get to a 92 test average.

**Example 2:** If the area of a trapezoid is 147 square inches and lengths of the two bases are 17 inches and 25 inches, what is the height of the trapezoid?

Solution:

We are asked to find the height of a trapezoid. It will be a good idea to sketch this situation.



From the formula list in the previous section, we know that  $A = \frac{1}{2}(b_1 + b_2)h$ .

We know these values:

$$A = 147$$

$$b_1 = 25$$

$$b_2 = 17$$

We can substitute these into the formula and solve for  $h$ :

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$147 = \frac{1}{2}(25 + 17)h$$

$$147 = \frac{1}{2}(42)h$$

$$147 = 21h$$

$$7 = h$$

We can conclude that the height of the trapezoid is 7 inches.

**Example 3:** Suppose you need to scan a homework assignment once each week for an online class. A neighborhood office service will scan your documents for \$1.50 per page. You can purchase a scanner for \$69.95 and learn to scan your own documents. How many pages will you need to scan in order for purchasing the scanner to be the better course of action?

Solution:

We want to know how many pages of scanning will equate the cost of purchasing a scanner.

let  $x$  = number of pages to scan

Now we can write our equation and solve it:

$$1.50x = 69.95$$

$$x \approx 46.63$$

If you will need to scan more than 46 pages, you should buy the scanner. The answer to the problem is 47 pages (or more).

**Example 4:** After dinner at a restaurant, you paid your bill and left a tip, for a total of \$52.52. If tax is 8.25% and you left a 20% tip, what was the menu cost of your food and beverages?

Solution:

We want to know the menu cost of your food and beverages.

let  $x$  = menu cost of food and beverages

The total bill is made up of three parts: the cost of your food and beverages as determined by the menu prices, tax on your meal and a tip. Tax will be 8.25% of the cost

of your food and beverages, and the amount of your tip will be 20% of your food and beverage cost. So we can write:

$$\text{Tax} = 0.0825x$$

$$\text{Tip} = 0.20x$$

Now we can write the equation and solve it:

$$x + 0.0825x + 0.20x = 52.52$$

$$1.2825x = 52.52$$

$$x = 40.95$$

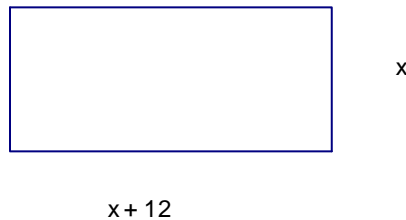
The menu cost of your food and beverages was \$40.95.

**Example 5:** Suppose your back yard is in the shape of a rectangle, with the length 12 feet more than the width. If it took 452 feet of fencing to enclose the yard, what are the dimensions of the yard?

Solution:

We first note that the problem involves a shape from geometry, so we should draw and label a rectangle. The problem tells us that the length is twelve feet more than the width, so we should let  $x$  = width. This will allow us to define both dimensions:

let  $x$  = width of the yard  
 $x + 12$  = length of the yard



We can use the formula for perimeter of a rectangle,  $P = 2l + 2w$ , since the amount of fencing needed to enclose the yard is the same as the perimeter.

$$P = 2l + 2w$$

$$452 = 2x + 2(x + 12)$$

$$452 = 2x + 2x + 12$$

$$452 = 4x + 24$$

$$428 = 4x$$

$$107 = x$$

We can conclude that the width of the yard is 107 feet. Since the length is 12 feet more than the width, we know that the length is  $107 + 12$  or 119 feet. So the dimensions of the yard are 107 feet by 119 feet.

**Example 6:** An isosceles triangle has two sides that are the same length. Suppose the perimeter of an isosceles triangle is 63 inches, and the two equal sides are each 4 times the length of the base. Find the lengths of the three sides of the triangle.

Solution:

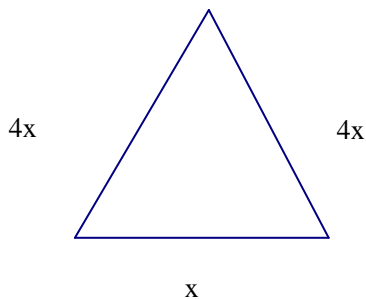
Start by defining a variable:

Let  $x$  = the length of the base.

If we know the length of the base, we can find the lengths of the other two sides.

$4x$  = length of each of the two equal sides.

Here's a sketch of the triangle:



To find the perimeter of a triangle, we add up the lengths of the sides. We also know that the perimeter is 63 inches. This gives us an equation,

$$x + 2(4x) = 63$$

We can solve the equation to find  $x$ .

$$x + 2(4x) = 63$$

$$x + 8x = 63$$

$$9x = 63$$

$$x = 7$$

The length of the base is 7 inches. The lengths of the other two sides are each  $4x$ , so each of the other two sides is 28 inches.

The lengths of the sides are 7 inches, 28 inches and 28 inches.

We can also use proportions to solve problems.

**Example 7:** A recipe that uses  $6\frac{1}{2}$  cups of sugar makes 120 cookies. Suppose you need to make 300 cookies. How much sugar will you need to make 300 cookies?

Solution:

The amount of sugar needed is proportional to the number of cookies baked, so we can set up a proportion to solve this problem.

Start by defining a variable:

Let  $x$  = the amount of sugar needed to make 300 cookies

Set up two fractions, so that the numerators contain the amount of sugar and the denominators contain the number of cookies to be made with that amount of sugar.

$$6\frac{1}{2} \text{ and } \frac{x}{300}$$

Write a proportion by setting these two fractions equal to one another:

$$\frac{6\frac{1}{2}}{120} = \frac{x}{300}$$

We can cross multiply to eliminate the fractions:

$$300 \cdot 6\frac{1}{2} = 120x$$

$$1950 = 120x$$

$$16.25 = x$$

The amount of sugar needed to make 300 cookies is 16.25 cups.