Section 3.7

Solving Inequalities

Earlier, we looked at two types of mathematical sentences, equations and inequalities. So far we have focused our attention on solving equations and on using equations to solve problems.

Now we'll shift our attention to inequalities.

The inequality symbols $<$ and $>$ establish that the relationship between two quantities is unequal – that is, that one quantity is larger than the other. We looked at these two symbols in our earlier work:

$<$ which is read "is less than"
$>$ which is read "is greater than"

In this section, we'll solve problems that include these and two other inequality symbols:

$\leq$ which is read "is less than or equal to"
$\geq$ which is read "is greater than or equal to".

These two symbols allow for the possibility that the two quantities on opposite sides of the symbol can be equal to one another.

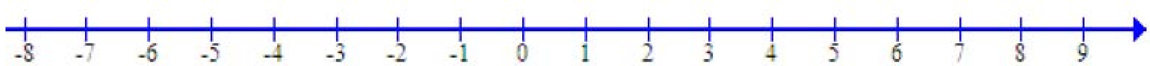
Graphing Inequalities on the Real Number Line

We can depict an inequality such as $x > 3$ using the real number line. This inequality means that x is the set of all real numbers that are larger than 3, not including 3 itself. We can show this on the real number line as an open circle at the number 3, and then we can shade all values to the right of 3 on the number line.

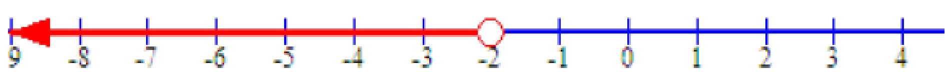
Example 1: Graph $x < -2$ on the real number line.

Solution:

We can draw and label a real number line:



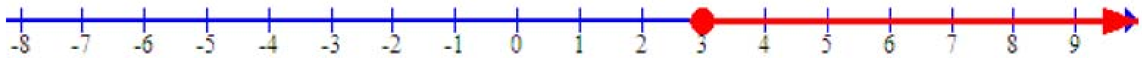
Since the graph will not contain -2, we'll draw an open circle at -2 and shade in all values to the left of -2 on the number line.



Example 2: Graph $x \geq 3$ on the real number line.

Solution:

We can draw and label a real number line. Since the graph contains 3, we'll draw a circle at 3 and shade it in, as well as all values to the right of 3 on the number line.

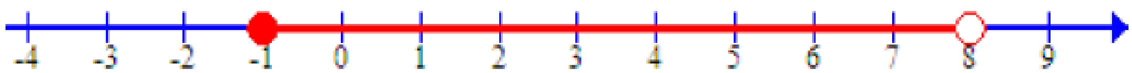


A **compound inequality** contains two inequality symbols. Here's a general example: $a < x \leq b$. This means that two statements are true: $x > a$ and $x \leq b$. On the number line, this will be all of the real numbers between *a* and *b*, including *b* but not including *a*.

Example 3: Graph $-1 \leq x < 8$ on the real number line.

Solution:

We can draw and label a real number line. Since the graph contains -1 and does not contain 8, we'll draw a circle at -1 and shade it in, we'll draw an open circle at 8 and then we'll shade all of the values in between the two circles on the number line.



Solving Inequalities

Many of the techniques for solving inequalities mimic similar techniques for solve equations with two very notable exceptions. Our objective is still to isolate the variable, and to do this, we can add the same quantity to both sides, subtract the same quantity from both sides, multiply both sides of the inequality by the same number or divide both sides of the inequality by the same number. However, if we need to multiply both sides of an inequality by a negative number or if we need to divide both sides of an inequality by a negative number, we must reverse the direction of the inequality sign.

Example 4: Solve the inequality. Graph the solution on the real number line: $-6x > 30$

Solution:

In this problem, we will need to multiply both sides of the equation by a negative number, so we'll need to reverse the direction of the inequality. Note that the problem involves a > symbol, but the answer involves a < symbol.

$$-6x > 30$$

$$\frac{-1}{6}\cdot -6x < 30\cdot\frac{-1}{6}$$

$$x < -5$$

Next we can graph the solution on the real number line:

Example 5: Solve the inequality. Graph the solution on the real number line:

$$2x - 7 \le 5$$

Solution:

We'll need to use two steps to solve the inequality, but we won't need to reverse the direction of the inequality symbol.

$$2x - 7 \le 5$$

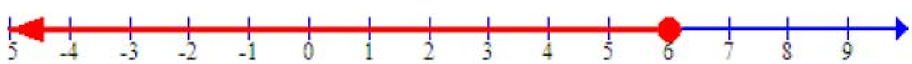
$$2x - 7 + 7 \le 5 + 7$$

$$2x \le 12$$

$$\frac{1}{2}\cdot 2x \le 12\cdot\frac{1}{2}$$

$$x \le 6$$

Now that we have the solution, we can graph it on the real number line:



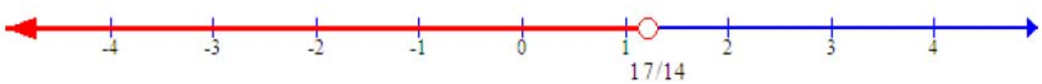
Some problems may contain fractions and/or parentheses. We'll work these as we worked equations, but we'll still need to reverse the direction of the inequality sign if we have to multiply or divide both sides of the inequality by a negative number.

Example 6: Solve the inequality: $3-4(2x-5)>3(2x-1)+9$. Graph the solution on the real number line.

Solution:

$$
\begin{aligned}
3-4(2x-5)&>3(2x-1)+9\\
3-8x+20&>6x-3+9\\
-8x+23&>6x+6\\
-8x+23-6x&>6x+6-6x\\
-14x+23&>6\\
-14x+23-23&>6-23\\
-14x&>-17\\
\frac{-1}{14}\cdot -14x&<-17\cdot\frac{-1}{14}\\
x&<\frac{17}{14}
\end{aligned}
$$

Now that we have the solution, we can graph it on the real number line.



Example 7: Solve the inequality: $\frac{1}{2}x-\frac{2}{3}(3-x)\le\frac{1}{4}(x-1)+5$. Graph the solution on the real number line.

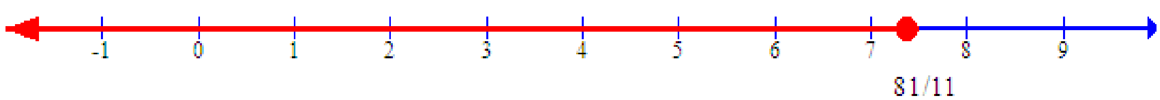
Solution:

We can start by finding the least common denominator for the three fractions in the problem. We can multiply both sides of the inequality by that number to clear it of fractions. Then we can solve the resulting inequality.

The LCD of 2, 3 and 4 is 12.

$$\frac{1}{2}x - \frac{2}{3}(3 - x) \le \frac{1}{4}(x - 1) + 5$$
$$12\left[\frac{1}{2}x - \frac{2}{3}(3 - x)\right] \le 12\left[\frac{1}{4}(x - 1) + 5\right]$$
$$6x - 8(3 - x) \le 3(x - 1) + 60$$
$$6x - 24 + 8x \le 3x - 3 + 60$$
$$14x - 24 \le 3x + 57$$
$$14x - 24 - 3x \le 3x + 57 - 3x$$
$$11x - 24 \le 57$$
$$11x - 24 + 24 \le 57 + 24$$
$$11x \le 81$$
$$\frac{1}{11} \cdot 11x \le \frac{1}{11} \cdot 81$$
$$x \le \frac{81}{11}$$
$$x \le 7\frac{4}{11}$$

Now that we know the solution, we can graph it on the real number line.



Applications Using Inequalities

We can use inequalities to help solve word problems.

Example 8: Two car rental agencies offer a summer special on their cars. Charles is planning to rent a car from one of the two agencies. Agency A offers a flat rate of $350 per week which includes unlimited mileage. Agency B rents the car for $275 per week and charges $0.40 per mile for each mile the customer drives. How many miles must he drive to make the unlimited mileage plan the better deal?

Solution:

If the unlimited mileage plan is to be the better deal, it must cost less than the plan offered by Agency B. We'll let x = the number of miles driven during the week.

We can express the cost to rent from Agency B as $275 + 0.40x$. We can write an inequality and then solve it:

$$350 < 275 + 0.40x$$
$$350 - 275 < 275 + 0.40x - 275$$
$$75 < 0.40x$$
$$\frac{75}{0.40} < \frac{0.40x}{0.40}$$
$$187.5 < x$$
$$x > 187.5$$

He must drive more 188 (or more) miles for the unlimited plan to be the better bargain.