# Section 4.1

# Graphing Linear Equations

In the last chapter, we solved equations that contained one unknown quantity – that is, one variable. Many problems involve more than one unknown quantity. For these problems, we will need to work with as many variables as there are unknowns. In this chapter, we'll look at equations that contain two variables. Although we can use any letters to stand for the variables, we'll most often use *x* and *y*.

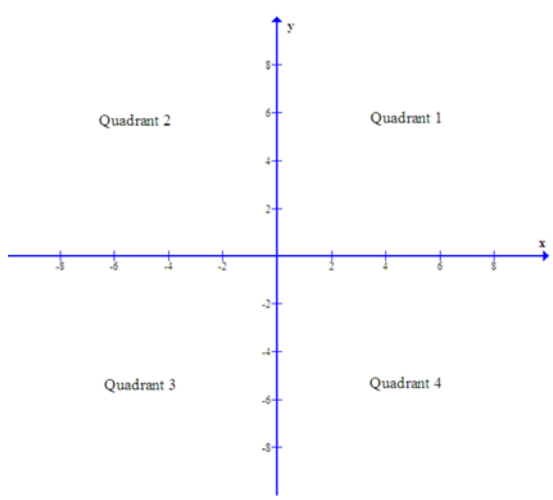
Here is an example of an equation with two variables: $x + y = 5$. A solution to this equation will consist of two numbers, one value for *x* and one value for *y*. We'll typically write a solution to an equation in two variables as an ordered pair $(x, y)$. You might notice that $x + y = 5$ has more than one solution. For example, $(0,5)$, $(1,4)$, $(2,3)$ and $(3,2)$ are all solutions to the equation – and there are many more. Note that (2, 3) and (3, 2) are not the same solution.

The set of all solutions to this – and any other – equation in two variables can be shown using a graph. We'll get to graphing lines soon, but first, here are some preliminaries.

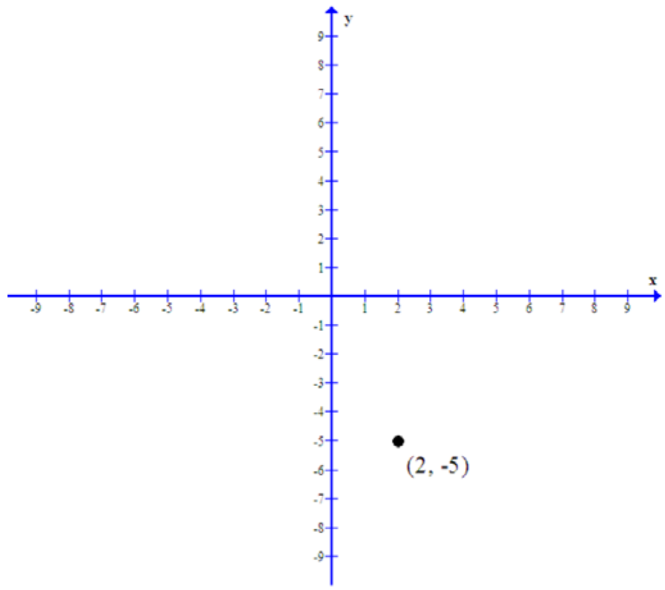
## The Cartesian Coordinate System

We'll use the Cartesian Coordinate System for graphing. This consists of two number lines that are perpendicular to one another. The horizontal number line is called the x axis, and the vertical number line is called the y axis. The two number lines intersect at the point (0, 0), which we call the origin. Numbers along the number lines that are to the right or above 0 are positive, and numbers that are to the left or below 0 on the number lines are negative numbers.

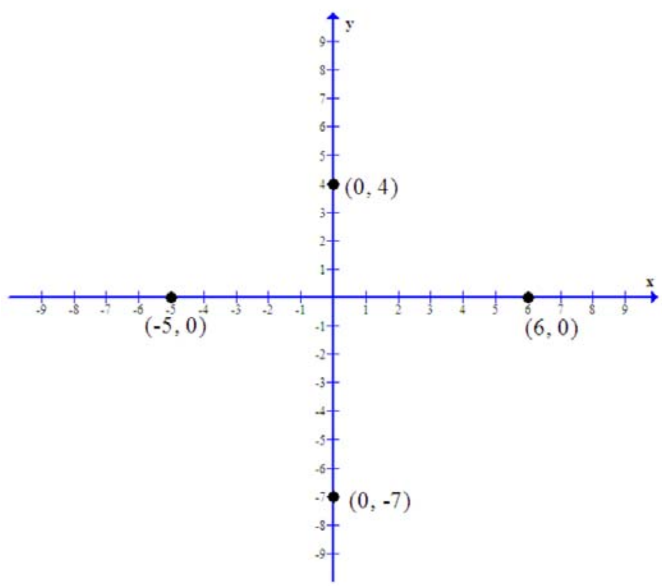
The coordinate plane is divided into four regions which we call quadrants. The upper right quadrant is referred to as Quadrant I. We then proceed counter-clockwise to name the rest of the quadrants.



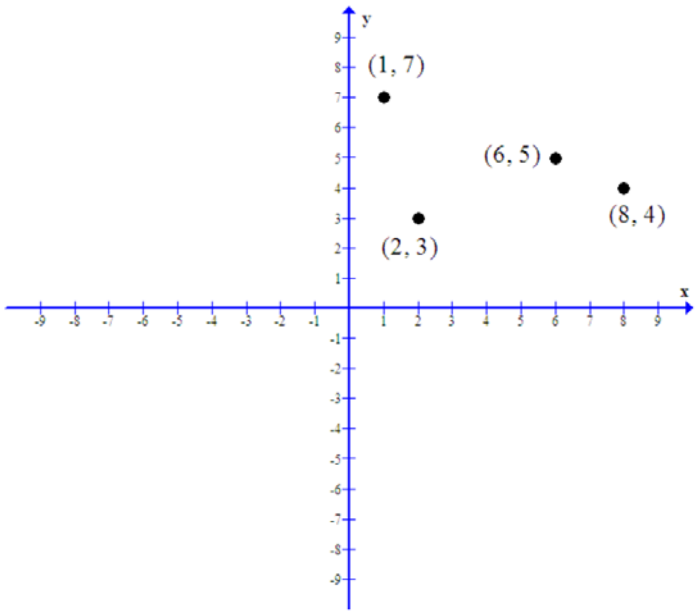
Each ordered pair (x, y) corresponds to a specific location in the coordinate plane. We graph points in the coordinate plane by starting at the origin, and then moving left or right according to the value for x. Then we move up or down according to the value for y. So for the ordered pair (2, -5), we'd start at the origin and then move two units to the right and then down 5.



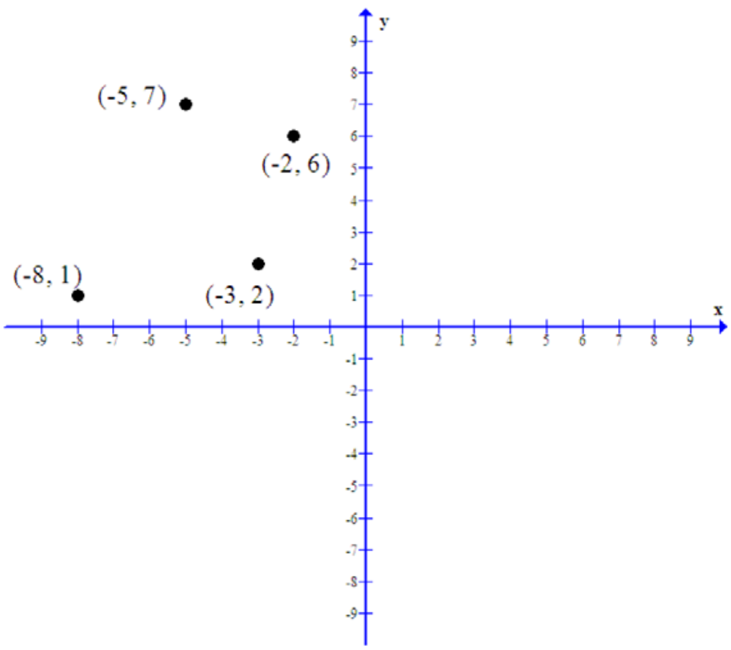

For an ordered pair with zero as one of the coordinates, the point will be located on one of the axes. If the *x* coordinate is 0, the point will lie on the *y* axis, and if the *y* coordinate is 0, the point will lie on the *x* axis.



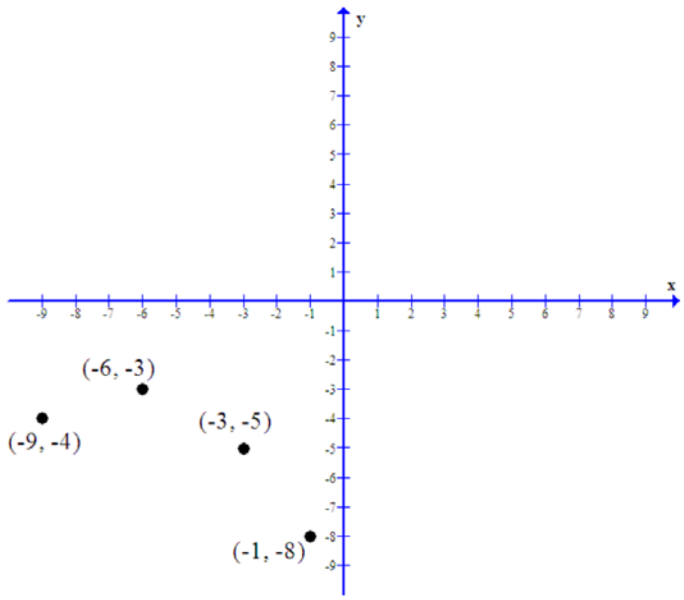
Ordered pairs with positive coordinates for both *x* and *y* will lie in the first quadrant.



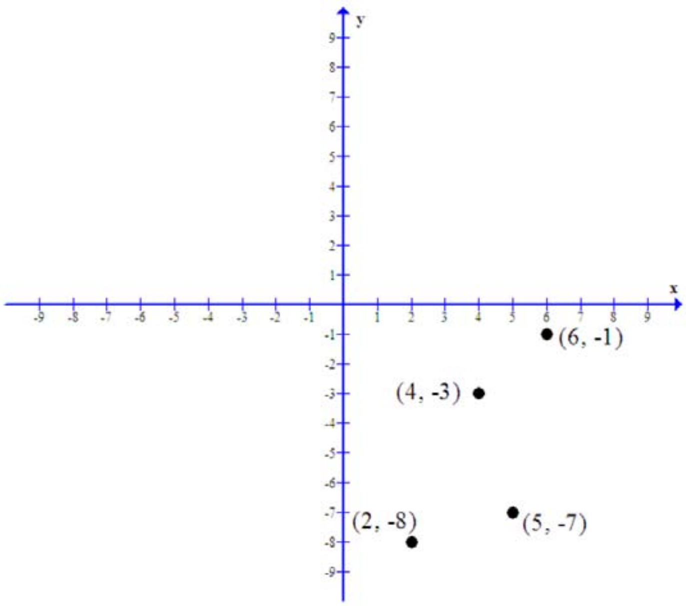

If the *x* coordinate is negative and the *y* coordinate is positive, the point will lie in Quadrant 2.




If both coordinates are negative, the point will lie in Quadrant 3.




If the *x* coordinate is positive and the *y* coordinate is negative, the point will lie in Quadrant 4.




## Graphing Lines

We need a minimum of two points to determine a line. We can use any of three methods for graphing lines in the coordinate plane: plotting points, using the method of intercepts, and using the slope and y-intercept to graph.

## Method 1: Plotting Points

For this method, we'll make a table of values, choosing numbers for one variable and then solving the equation to find the other variable. Once we have a table of values, we'll plot the points in the coordinate plane and then connect them with a line. We'll need a minimum of two points. If you find more points, though, you may find the graphing to be a little easer.

**Example 1:** Make a table of values and then plot the points you find in the coordinate plane to sketch the line $y = 2x - 3$.

Solution:

First, we'll construct a table of values and choose some values for x:

| $x$ | $y$ |
|---|---|
| -1 | |
| 0 | |
| 1 | |

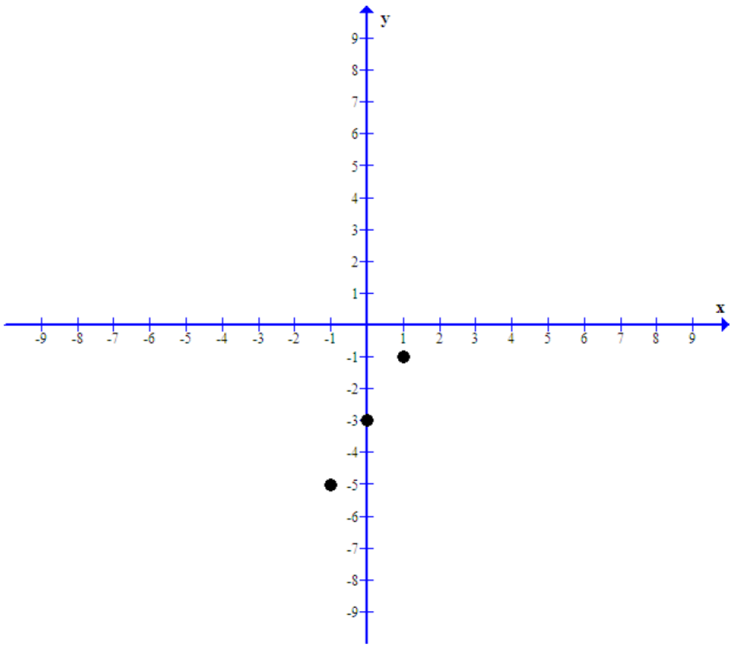
Now, we'll need to find the y values.

$$y = 2(-1) - 3 = -2 - 3 = -5$$
$$y = 2(0) - 3 = 0 - 3 = -3$$
$$y = 2(1) - 3 = 2 - 3 = -1$$

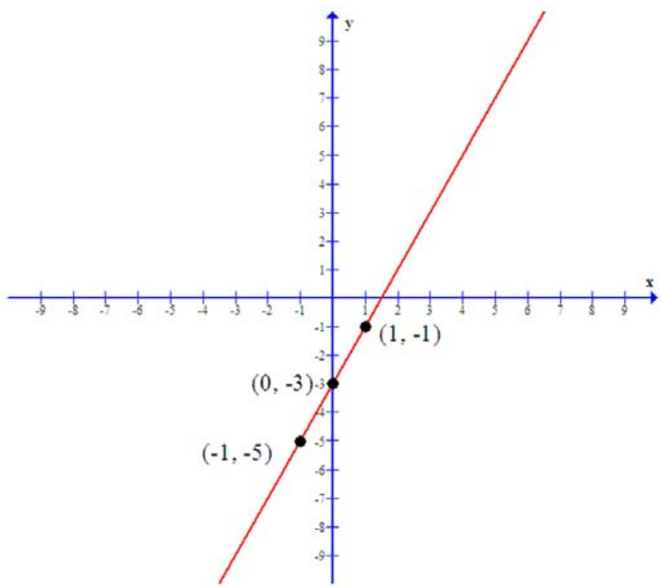
We can fill these values into the y column of the table of values.

| $x$ | $y$ |
|---|---|
| -1 | -5 |
| 0 | -3 |
| 1 | -1 |

We've found three ordered pairs, (-1, -5), (0, -3) and (1, -1) which we can plot in the coordinate plane.




Finally, we'll use the points as a guide to draw in the line.



**Example 2:** Make a table of values and then plot the points you find in the coordinate plane to sketch the line $2x - 3y = 6$.

Solution:

First, we'll construct a table of values and choose some values for x:

| $x$ | $y$ |
|---|---|
| -3 | |
| 0 | |
| 3 | |

Now, we'll need to find the y values.

$2(-3) - 3y = 6$

$-6 - 3y = 6$

$-3y = 12$

$y = -4$

So we have a point (-3, -4).

$2(0) - 3y = 6$

$0 - 3y = 6$

$-3y = 6$

$y = -2$

We've found another point, (0, -2).

$2(3) - 3y = 6$

$6 - 3y = 6$

$-3y = 0$

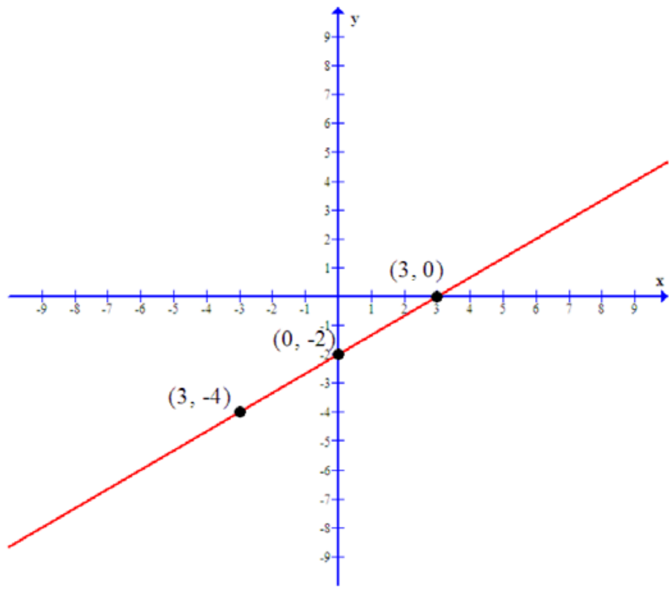
$y = 0$

This gives us a third point (3, 0).

We can fill these values into the y column of the table of values.

| $x$ | $y$ |
|---|---|
| -3 | -4 |
| 0 | -2 |
| 3 | 0 |

Now we can use these points to graph the line.




## Method 2: Using the intercepts to graph

This method is really just a variation of method 1. Instead of choosing just any points, we'll choose two very specific points: the *x* intercept and the *y* intercept.

**Example 3:** Use the method of intercepts of graph $2x+5y=10$.

Solution:

This time, we'll just find two points, the one where *y* is zero and the one where *x* is zero.

| $x$ | $y$ |
|---|---|
| | 0 |
| 0 | |

To find the *x* intercept, we'll let y = 0 and solve for *x*.

$$2x+5y=10$$
$$2x+5(0)=10$$
$$2x=10$$
$$x=5$$

So the x intercept is (5, 0).

Now we'll find the *y* intercept. Let *x* = 0 and solve for *y*.

$2x+5y=10$

$2(0)+5y=10$

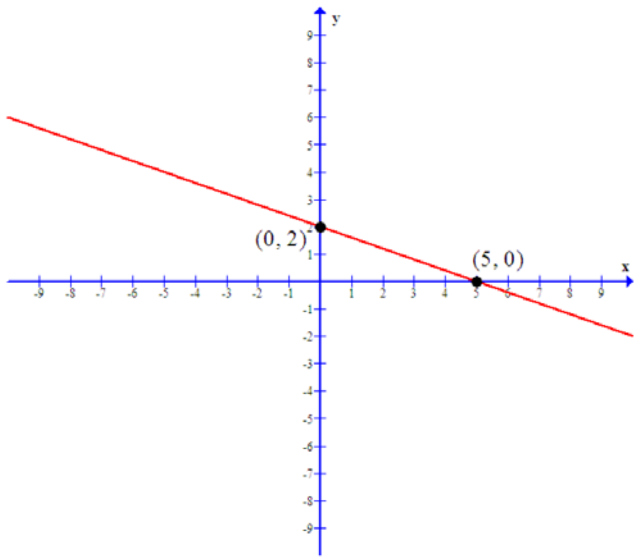
$5y=10$

$y=2$

So the y intercept is (0, 2).

| $x$ | $y$ |
|---|---|
| 5 | 0 |
| 0 | 2 |

Finally, we'll use these two points to graph the line.




**Example 4:** Use the method of intercepts of graph $3x-4y=8$.

Again, we'll let y = 0 and solve for x, and then we'll let x = 0 and solve for y.

| $x$ | $y$ |
|---|---|
| | 0 |
| 0 | |

$3x-4y=8$

$3x-4(0)=8$

$3x-0=8$

$3x=8$

$x=8/3=2\frac{2}{3}$

So the x intercept is $\left(2\frac{2}{3},0\right)$.

$3(0)-4y=8$

$0-4y=8$

$-4y=8$

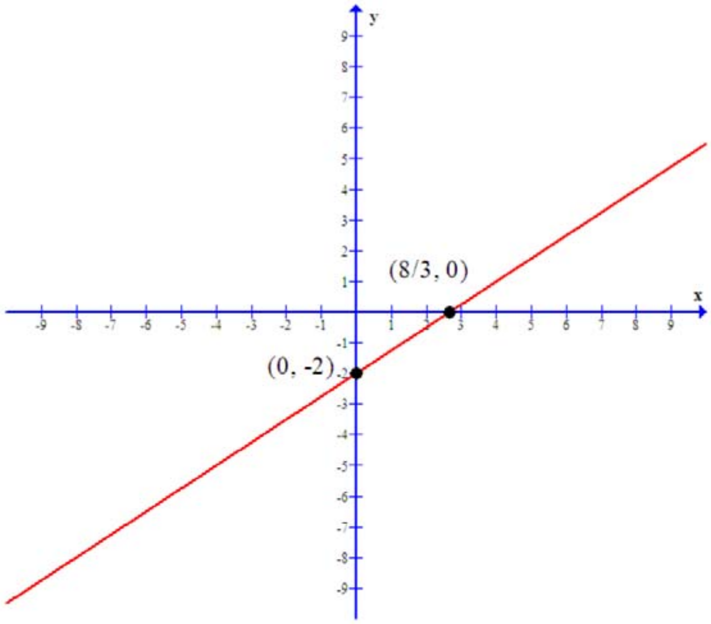
$y=-2$

We've found that the y intercept is (0, -2)

Here’s the table of values:

| $x$ | $y$ |
|---|---|
| $2\frac{2}{3}$ | 0 |
| 0 | -2 |

Now we’ll use these points to graph the line.



## Method 3:  Using the slope and y intercept to graph.

Before we can tackle this method for graphing, we need to cover a few preliminaries.

**Slope of a Line**

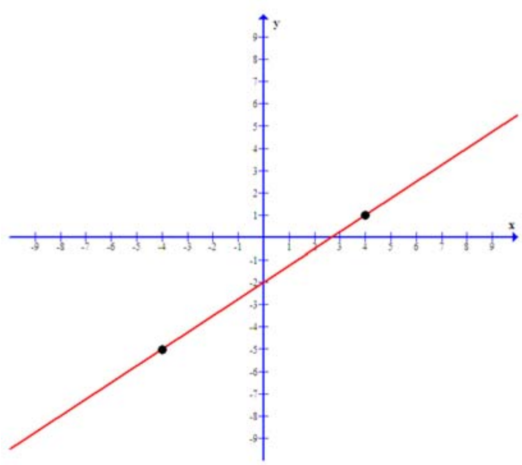
The slope of a line is its rate of change.  It is a measure of the "steepness" of the line. Slope can be positive, negative, zero or undefined.  If a line is moving upward as you look from left to right, the line has a positive slope.  If a line is moving downward as you look from left to right, the line has a negative slope.  If the line is horizontal, the slope is zero, and if the line is vertical, the slope is undefined.  The closer a line is to being horizontal, the smaller its slope; the closer a line is to being vertical, the larger its slope.

So a line with a slope of 9 is very steep, while a line with a slope of $\frac{1}{10}$ will be nearly horizontal.

Slope can be defined mathematically in several ways.

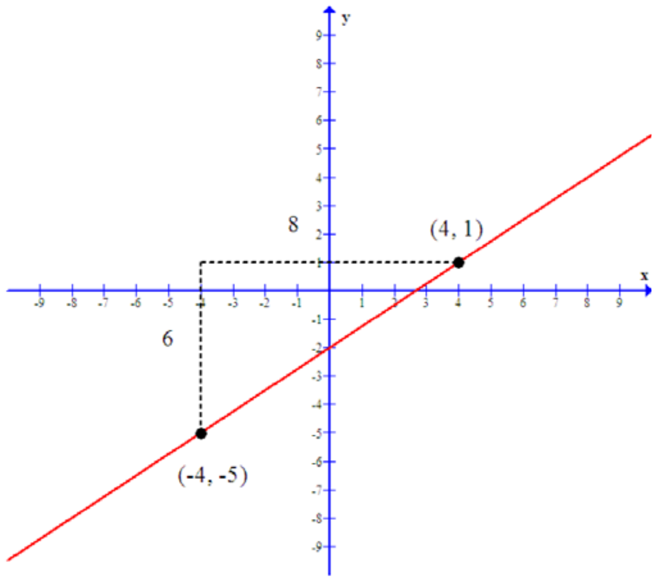
We define slope as $\frac{\text{change in y}}{\text{change in x}}$ or $\frac{\text{rise}}{\text{run}}$.  If we know two points $(x_{1,}y_1)$ and $(x_{2,}y_2)$ that lie on the line, we can compute the slope using the slope formula:  $slope = m = \frac{y_2 - y_1}{x_2 - x_1}$.

**Example 5:**  Find the slope of the line that is graphed.



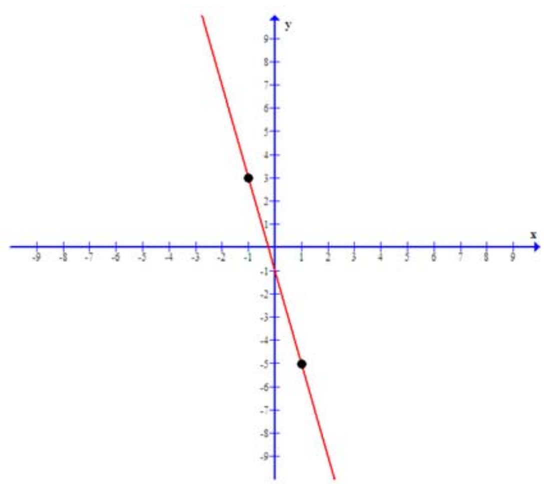

Solution:

In the example, we can find the slope by counting.  There are two prominent points on this graph, (-4, -5) and (4, 1).  We want to start at (-4, -5) and draw a right triangle that will allow us to move first up and then right to land at (4, 1).  Then we can count the "rise" and the "run.




In this case, we'll need to rise 6 units and run 8 units, so the slope is $\frac{3}{4}$.

**Example 6:** Find the slope of the line that is graphed.




Solution:

We can identify two points, (1, -5) and (-1, 3).  To get from (-1, 3) to (1, -5), we'll need to move down eight units and then move two unit to the right.  So the slope of the line is $\frac{-8}{2} = -4$.

Note that in both of these examples, we could have used the slope formula to find the slope of the line.

**Example 7:** Use the slope formula to find the slope of the line that passes through the points (-1, 5) and (2, -4).

Solution:

In this example, we have

$$x_1 = -1$$
$$y_1 = 5$$
$$x_2 = 2$$
$$y_2 = -4$$

Substitute these values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ and evaluate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{-4-5}{2-(-1)} = \frac{-9}{3} = -3$$

The slope is -3.

**Example 8:** Use the slope formula to find the slope of the line that passes through the points (7, -1) and (-2, 6).

Solution:

In this example, we have

$$x_1 = 7$$
$$y_1 = -1$$
$$x_2 = -2$$
$$y_2 = 6$$

Substitute these values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ and evaluate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{6-(-1)}{-2-7} = \frac{7}{-9} = -\frac{7}{9}$$

The slope is $-\frac{7}{9}$.

**Example 9:** Use the slope formula to find the slope of the line that passes through the points (6, 4) and (-1, 4).

Solution:

In this example, we have

$$x_1 = 6$$
$$y_1 = 4$$
$$x_2 = -1$$
$$y_2 = 4$$

Substitute these values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ and evaluate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{4-4}{-1-6} = \frac{0}{-7} = 0$$

The slope is 0.

**Example 10:** Use the slope formula to find the slope of the line that passes through the points (5, -3) and (5, 7).

Solution:

In this example, we have

$$x_1 = 5$$
$$y_1 = -3$$
$$x_2 = 5$$
$$y_2 = 7$$

Substitute these values into the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$ and evaluate.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7-(-3)}{5-5} = \frac{10}{0}$$

Division by zero is not defined, so the slope is undefined.

**The Slope-Intercept Form of the Equation of the Line**

An equation of the form $y = mx + b$ is called the slope-intercept form of the equation of the line. We can easily find the slope of the line and the y-intercept just by looking at the equation. The slope is *m* and the *y* intercept is the point $(0, b)$.

**Example 11:** Suppose $y = \frac{2}{5}x - 6$. Identify the slope and the y intercept.

Solution:

Compare the equation with the form $y = mx + b$. In this case, $m = \frac{2}{5}$ and $b = -6$, so the slope is $\frac{2}{5}$ and the y intercept is the point (0, -6).

Sometimes an equation will be given in the general form, $Ax + By = C$. In that case, you will need to start by rewriting the equation in the form, $y = mx + b$. Then you can read off the slope and the y intercept.

**Example 12:** Suppose $5x - 2y = 6$. Write the equation in slope-intercept form. Then state the slope and the y intercept.

Solution:

Start by solving the equation for y:

$$5x - 2y = 6$$
$$-2y = -5x + 6$$
$$y = \frac{5}{2}x - 3$$

Now state the slope and the y intercept:

$m = \frac{5}{2}$ and $b = -3$

So the slope is $\frac{5}{2}$ and the y intercept is (0, -3).

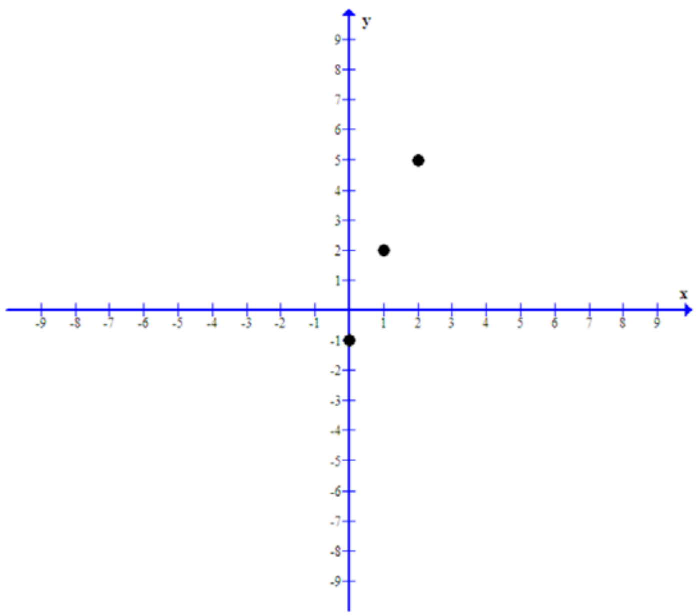
Now we can get back to graphing.

To graph using the slope and y intercept, begin by plotting the y intercept. Then use the slope to graph a second point, starting at the y intercept and counting the rise and the run. If you'd like to, you can graph a third point from the second point. Then connect the points to draw in the line.

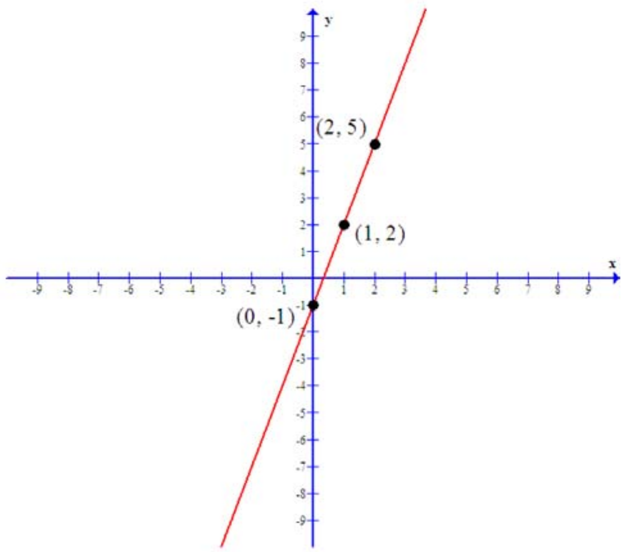
**Example 13:** Suppose $y = 3x - 1$. State the slope and the y intercept. Then use them to graph the line.

Solution:

In this example, $m = 3$ and $b = 1$, so the slope is 3 and the y intercept is (0, -1). Since we need to graph the line, we'll write the slope as a fraction, $m = \frac{3}{1}$.
Start by graphing the y intercept, then use the slope to find another point or two.




Next, connect the points with a line.

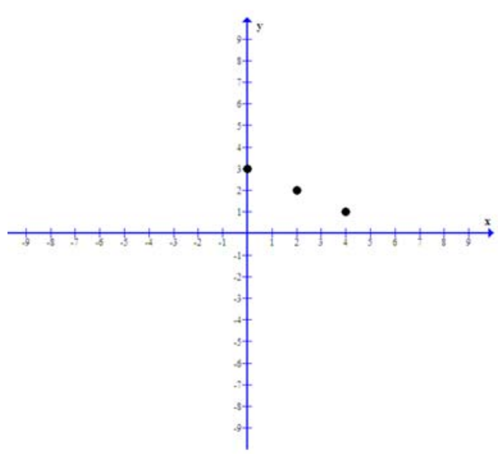



**Example 14:** Graph using the slope and the y intercept: $x+2y=6$

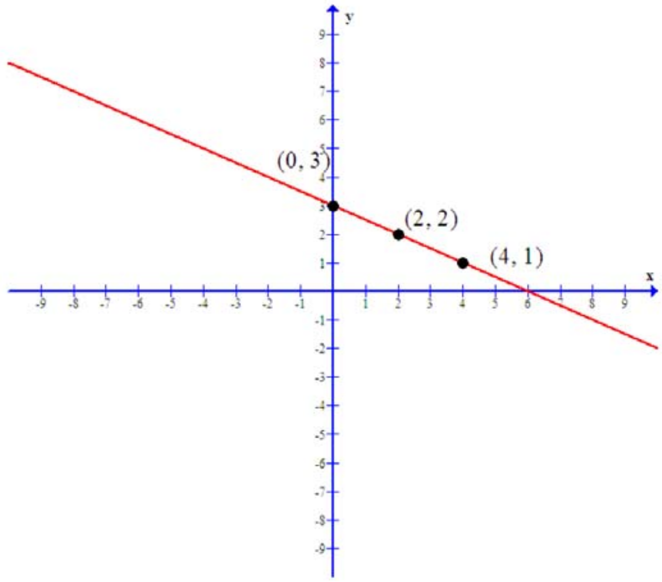
First, write the equation in the form $y=mx+b$.

$$x+2y=6$$
$$2y=-x+6$$
$$y=-\frac{1}{2}x+3$$

Slope is $\frac{-1}{2}$ and the y intercept is (0, 3). Now graph using the slope and the y intercept. Plot the y intercept and then use the slope (rise = -1, run = 2) to graph a couple of other points.




Now connect the dots.

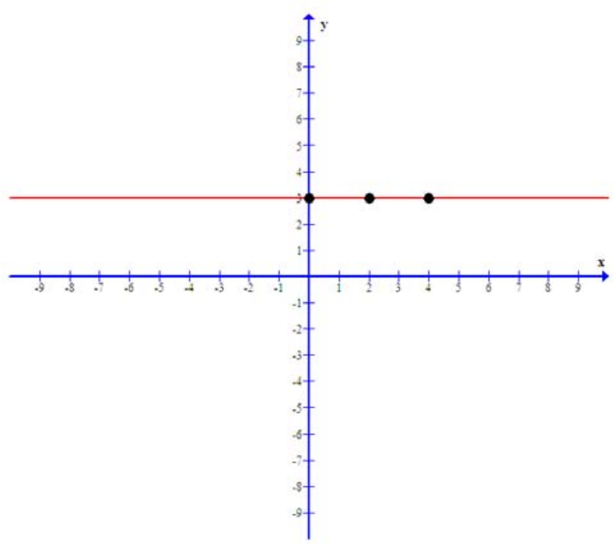



## Graphing Vertical and Horizontal Lines

**Example 15:** Graph $y = 3$.

Solution:

The graph of this line will pass through the point (0, 3) and will have slope 0. All of the points on the line will have a y coordinate of 3.



**Example 16:** Graph $x = -2$.

Solution:

The graph is a vertical line that passes through the point (-2, 0). All of the points on the line will have an x coordinate of -2.