

## Section 4.2

# Writing Equations of Lines

In some examples, you'll be given information about the graph of a function and you'll be asked to write the equation of the line. In this section, we'll work through some examples of each type of problem you'll be expected to work. You'll write your final answers in slope-intercept form, that is,  $y = mx + b$ .

## Writing Equations of Lines

**Example 1:** Write an equation of the line that has slope -4 and y intercept (0, 9).

Solution:

Since we know both the slope and the y intercept, we can just substitute -4 for  $m$  and 9 for  $b$  into the slope intercept form:

$$y = mx + b$$

$$y = -4x + 9$$

In many of these examples, we'll use the point-slope form of the equation of the line,  $y - y_1 = m(x - x_1)$ , where  $m$  is the slope and  $(x_1, y_1)$  is a point that lies on the line

**Example 2:** Write an equation of the line that has slope  $\frac{1}{3}$  and passes through the point (-6, 4).

Solution:

In this problem, we have these values:

$$m = \frac{1}{3}$$

$$x_1 = -6$$

$$y_1 = 4$$

Start by substituting them into the point-slope form.

$$y - 4 = \frac{1}{3}(x - (-6))$$

Now rewrite this equation in slope intercept form.

$$y - 4 = \frac{1}{3}(x - (-6))$$

$$y - 4 = \frac{1}{3}(x + 6)$$

$$y - 4 = \frac{1}{3}x + 2$$

$$y = \frac{1}{3}x + 6$$

**Example 3:** Write an equation of the line that passes through the points (4, -6) and (1, 3).

Solution:

In this example, we must first find the slope. We have the following values:

$$x_1 = 4$$

$$y_1 = -6$$

$$x_2 = 1$$

$$y_2 = 3$$

We can substitute these into the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - (-6)}{1 - 4} = \frac{3 + 6}{-3} = \frac{9}{-3} = -3$$

Now that we have the slope, we can proceed as we did in Example 2 using the point-slope formula. We can use either one of the points for  $(x_1, y_1)$ .

$$y - (-6) = -3(x - 4)$$

$$y + 6 = -3x + 12$$

$$y = -3x + 6$$

**Example 4:** Write an equation of the line that passes through the points (4, 9) and (4, -7).

Solution:

In this example, we have the following values:

$$x_1 = 4$$

$$y_1 = 9$$

$$x_2 = 4$$

$$y_2 = -7$$

First, we'll find the slope using these values:

$$m = \frac{-7 - 9}{4 - 4} = \frac{-16}{0}$$

Since division by zero is not defined, we know that the slope is undefined. This tells us that the line will be a vertical line and will pass through both of the points. Notice that both of the points have an  $x$  coordinate of 4. So the equation of the line will be  $x = 4$ .

**Example 5:** Write an equation of the line that passes through the points (2, 1) and (-6, 1).

In this example, we have the following values:

$$x_1 = 2$$

$$y_1 = 1$$

$$x_2 = -6$$

$$y_2 = 1$$

First, we'll find the slope using these values:

$$m = \frac{1 - 1}{-6 - 2} = \frac{0}{-8} = 0$$

Since the slope is 0, we know that the line will be a horizontal line and will pass through both of the points. Notice that both of the points have an  $y$  coordinate of 1. So the equation of the line will be  $y = 1$ .

We could also use the point-slope form, using the slope and one of the points, and we'll reach the same result:

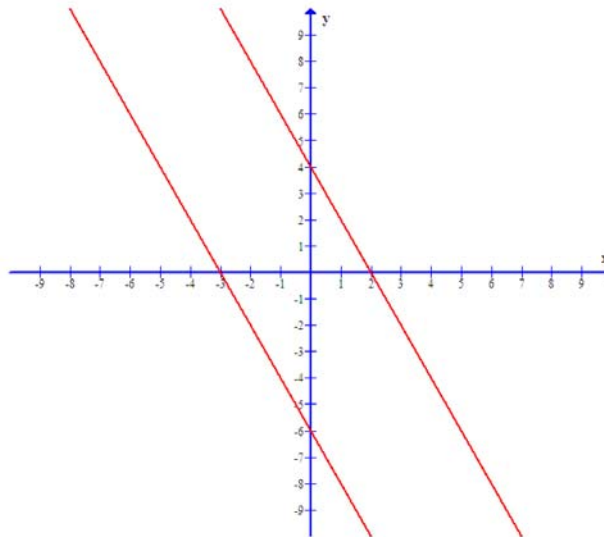
$$y - 1 = 0(x - 2)$$
$$y - 1 = 0$$
$$y = 1$$

## Parallel and Perpendicular Lines

We'll also write equations of lines that are parallel to or perpendicular to a given line and which pass through a specified point.

Two lines are parallel if they have the same slope. So the lines  $y = -2x + 4$  and  $y = -2x - 6$  since both lines have  $-2$  as the slope.

Graphs of the two lines are shown here:

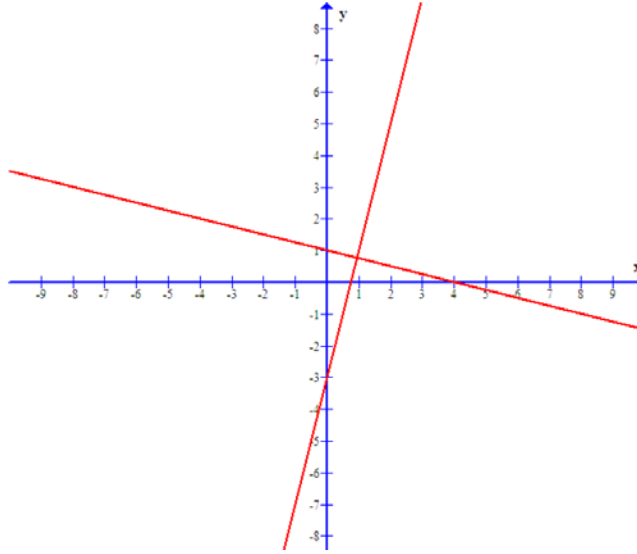


Two lines are perpendicular if their slopes are negative reciprocals; that is if the slope of one line is  $m$  and the slope of another line is  $\frac{-1}{m}$ , then the two lines are perpendicular.

You also know that two lines are perpendicular if the product of their slopes is  $-1$

So the lines  $y = \frac{-1}{4}x + 1$  and  $y = 4x - 3$  are perpendicular since  $\frac{-1}{4} \cdot 4 = -1$ . The two lines will intersect at a right angle.

Graphs of the two lines are shown here:



We'll be interested in writing equations of lines.

**Example 6:** Write an equation of the line that is parallel to  $y = -\frac{1}{2}x + 5$  and passes through the point (4, 2).

Solution:

The slope of the line  $y = -\frac{1}{2}x + 5$  is  $-\frac{1}{2}$ . Since our equation needs to give the line that is parallel to the given line, it will have the same slope. We can use the point-slope form of the equation of the line to find the equation:

$$y - 2 = -\frac{1}{2}(x - 4)$$

$$y - 2 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 4$$

**Example 7:** Write an equation of the line that is parallel to  $2x - 3y = 12$  and passes through the point (-6, -1)

Solution:

In this example, the slope is not obvious. We must first write the given equation in the form  $y = mx + b$  so that we can easily find the slope:

$$\begin{aligned}
 2x - 3y &= 12 \\
 -3y &= -2x + 12 \\
 y &= \frac{2}{3}x - 4
 \end{aligned}$$

Now we can see that the slope of the line is  $\frac{2}{3}$ . We'll write an equation of the line whose slope is  $\frac{2}{3}$  and passes through the point  $(-6, -1)$ :

$$\begin{aligned}
 y - (-1) &= \frac{2}{3}(x - (-6)) \\
 y + 1 &= \frac{2}{3}(x + 6) \\
 y + 1 &= \frac{2}{3}x + 4 \\
 y &= \frac{2}{3}x + 3
 \end{aligned}$$

**Example 8:** Write an equation of the line that is perpendicular to the line  $y = \frac{2}{5}x - 7$  and passes through the point  $(-4, 5)$ .

Solution:

The slope of the given line is  $\frac{2}{5}$ , so a line perpendicular to this line will have slope  $-\frac{5}{2}$ . We'll use this slope and the point  $(-4, 5)$  to write an equation of the line.

$$\begin{aligned}
 y - 5 &= -\frac{5}{2}(x - (-4)) \\
 y - 5 &= -\frac{5}{2}(x + 4) \\
 y - 5 &= -\frac{5}{2}x - 10 \\
 y &= -\frac{5}{2}x - 5
 \end{aligned}$$

**Example 9:** Write an equation of the line that is perpendicular to the line that passes through the points (1, 5) and (4, 9) and passes through the point (-8, 2).

Solution:

We are not given the equation of a line this time, but we know two points that the line contains, so we can find its slope. We have these values given:

$$x_1 = 1$$

$$y_1 = 5$$

$$x_2 = 4$$

$$y_2 = 9$$

We'll find the slope of the given line:

$$m = \frac{9-5}{4-1} = \frac{4}{3}$$

We want the equation of the line that is perpendicular to this line, so the slope of our line will be  $-\frac{3}{4}$ . We'll use this slope with the specified point (-8, 2) to write the equation of the line:

$$y - 2 = -\frac{3}{4}(x - (-8))$$

$$y - 2 = -\frac{3}{4}(x + 8)$$

$$y - 2 = -\frac{3}{4}x - 6$$

$$y = -\frac{3}{4}x - 4$$