

Section 4.3

Graphing Linear Inequalities

We'll also solve some linear inequalities in two variables. For these problems, the solution will be a graph. In each instance, we'll graph a line as either a solid line or a dotted line, depending on the problem, and then shade the half-plane on the appropriate side.

For problems containing the symbols \leq or \geq , the line we'll draw will be a solid line, since these inequalities include all of the points on the line. For problems containing the symbols $<$ or $>$, we'll draw a dotted line, since these inequalities will include all points up to, but not including, the line.

Determining if a Point Satisfies an Inequality

To determine if a point satisfies an inequality, substitute in the values for x and y and see if the resulting mathematical sentence is true.

Example 1: Determine if the ordered pair $(1, -6)$ satisfies the inequality $2x - 3y > 15$.

Solution:

Let $x = 1$ and $y = -6$.

Then we have $2(1) - 3(-6) = 2 + 18 = 20$.

Since $20 > 15$ is a true statement, $(1, -6)$ satisfies the inequality.

Example 2: Determine if the ordered pair $(3, 5)$ satisfies the inequality $y \leq 2x - 4$.

Solution:

Let $x = 3$ and $y = 5$.

Then we have

$$5 \leq 2(3) - 4$$

$$5 \leq 6 - 4$$

$$5 \leq 2$$

Since $5 \leq 2$ is a false statement, $(3, 5)$ does not satisfy the inequality.

Example 3: Determine if the point $(2, -7)$ satisfies the inequality $7x + 3y \leq -7$.

Solution:

Let $x = 2$ and $y = -7$.

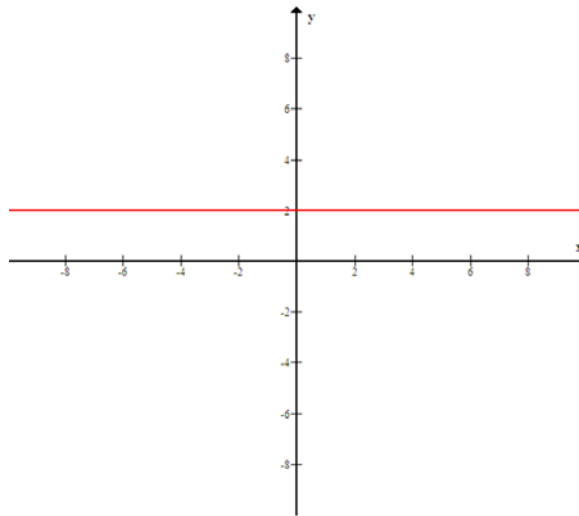
Then we have $7(3) + 2(-7) = 21 - 14 = -7$

Since $-7 \leq -7$ is a true statement, $(2, -7)$ satisfies the inequality.

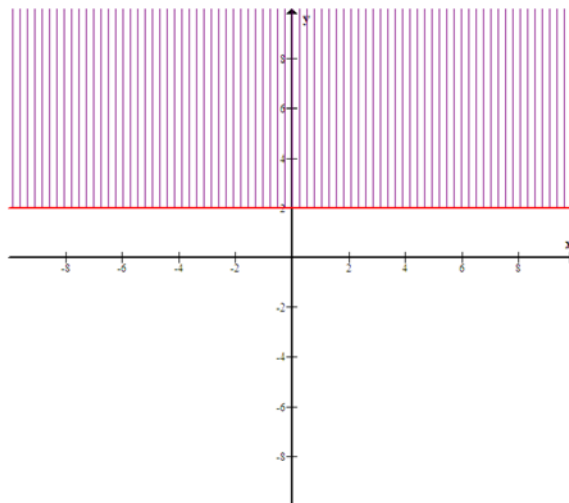
Example 4: Solve by graphing: $y \geq 2$.

Solution:

We'll start by graphing the line $y = 2$. This is a horizontal line that passes through all of the points that have y values of 2. The line will be a solid line since the inequality symbol is \geq .



We want the set of points with y values that are greater than 2, so we'll shade the set of points above the line.

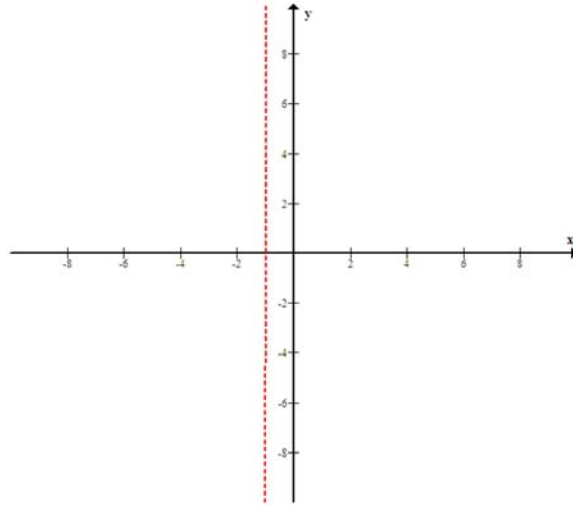


The graph is the solution.

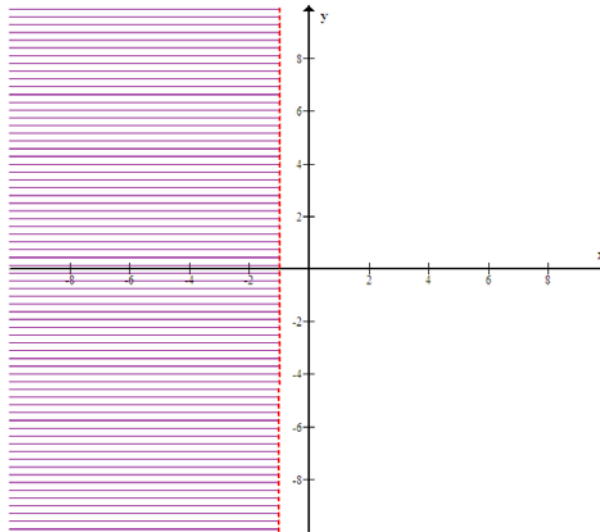
Example 5: Solve by graphing: $x < -1$.

Solution:

We'll start by graphing the line $x = -1$. This is a vertical line through all of the points with x values of -1 . The line will be a dotted line, since the problem contains the inequality symbol $<$ which does not include the points on the line.



Since we want all points where $x < -1$, we'll shade in all of the points to the left of the line.

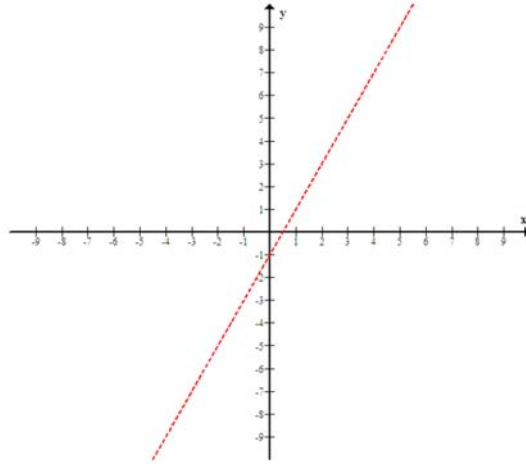


Now we'll tackle some problems that do not involve horizontal or vertical lines. Our method will be quite similar, but we'll have to include a test to determine which side of the line to shade.

Example 6: Solved by graphing: $y > 2x - 1$

Solution:

First we'll graph the line $y = 2x - 1$ as a dotted line. The slope is 2 and the y intercept is -1.



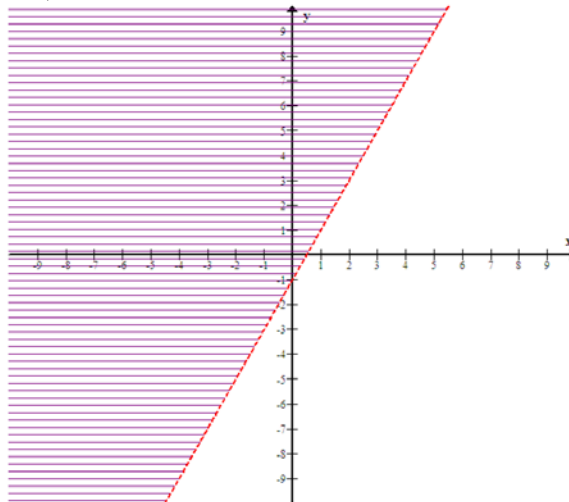
Next, pick a point that does not lie on the line. The origin is a convenient point to use, so long as the line does not pass through the origin. In this case, we can use the origin as our test point. See if the test point satisfies the inequality:

$$0 > 2(0) - 1$$

$$0 > 0 - 1$$

$$0 > -1$$

This is a true statement, so $(0, 0)$ satisfies the inequality. That tells us that $(0, 0)$ is in the solution, and is on the side of the line that should be shaded. To finish the problem, we'll shade the side where $(0, 0)$ lies:



Example 7: Solve by graphing: $3x - 4y \geq 12$

Solution:

We'll start by graphing the line $3x - 4y = 12$. The graph of this line will be a solid line since the problem contains the inequality symbol \geq .

An easy way to graph the line will be to find the x and y intercepts of the graph of the line:

x	y
0	0

Letting $y = 0$, we have

$$3x - 4(0) = 12$$

$$3x = 12$$

$$x = 4$$

Now letting $x = 0$, we have

$$3(0) - 4y = 12$$

$$0 - 4y = 12$$

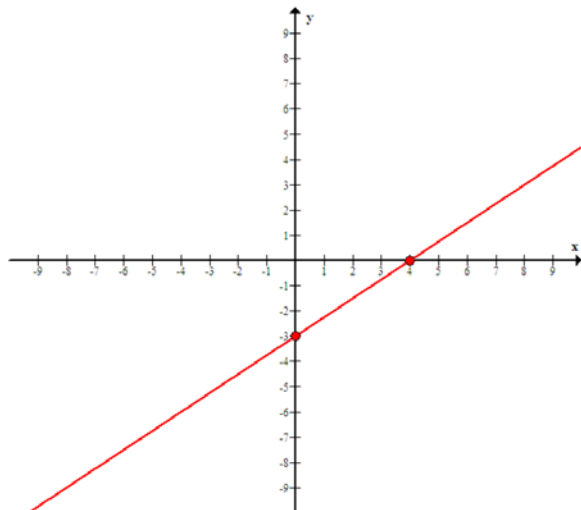
$$-4y = 12$$

$$y = -3$$

So we can complete the table:

x	y
4	0
0	-3

Next, we'll plot these two points and use them to graph the line:



Now we'll choose a test point. The line does not pass through the origin, so we can use $(0, 0)$ as our test point. We'll see if it satisfies the inequality:

$$3(0) - 4(0) \geq 12$$

$$0 \geq 12$$

Since $0 \geq 12$ is a false statement, we know that $(0, 0)$ does not lie in the solution of the inequality. This means we need to shade the half-plane that does not include the point $(0, 0)$, that is the set of points below the line.

The solution to the problem is the set of points on or below the line.

