Section 5.2

Linear Functions

We have already graphed lines and written equations of lines. When we graph a line – any line, except for a vertical line – the line will pass the vertical line test. So every non-vertical line can be expressed as a linear function. We usually write $y = mx + b$, but when we talk about an equation of a line as a linear function, we'll replace *y* with $f(x)$. So $f(x) = mx + b$ represents a linear function.

Working with Linear Functions

Just as with an equation of the form $y = mx + b$, we can find the slope and the *y* intercept of a linear function. We can also find the *x* intercept of the function, and we can use function notation to evaluate the function at various values of the variable.

Example 1: Suppose $f(x) = 0.25x + 1.4$. Find the slope, the *x* intercept, the *y* intercept and $f(16)$. Graph the linear function.

Solution:

Since the function is in the form $f(x) = mx + b$, we can read off the slope and the *y* intercept:

m = 0.25
y intercept is (0, 1.4)

To find the *x* intercept, we'll let $f(x) = 0$ and solve for *x*.

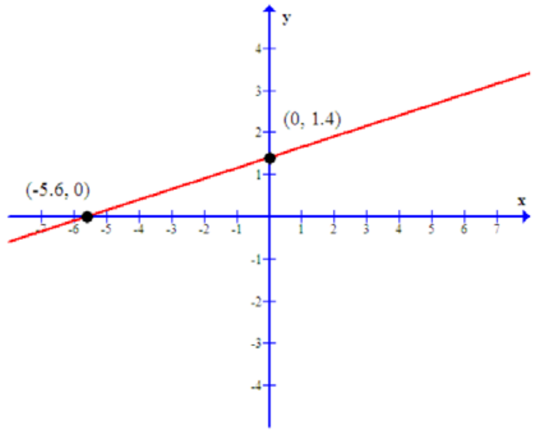
$$\begin{aligned} 0 &= 0.25x + 1.4 \\ -1.4 &= 0.25x \\ \frac{-1.4}{0.25} &= x \\ -5.6 &= x \end{aligned}$$

The *x* intercept is the point (-5.6, 0).

To find $f(16)$, we'll substitute 16 into the function and evaluate:

$$\begin{aligned} f(16) &= 0.25(16) + 1.4 \\ &= 4 + 1.4 \\ &= 5.4 \end{aligned}$$

We can use the *x* and *y* intercepts to graph the linear function:



Applications

We can use linear functions to help us solve a variety of problems.

Example 2: The formula $F(C) = \frac{9}{5}C + 32$ gives temperature in degrees Fahrenheit as a function of degrees Celsius. If the temperature is 38 degrees Celsius, what is the temperature on the Fahrenheit scale?

Solution:

We need to compute $F(38)$:

$$F(C) = \frac{9}{5}C + 32$$

$$\begin{aligned} F(38) &= \frac{9}{5}(38) + 32 \\ &= 68.4 + 32 \\ &= 100.4 \end{aligned}$$

The temperature is $100.4°F$.

Example 3: A company produces high end cell phones. Their monthly costs to produce the phones include $225,000 in overhead costs and $69 in material costs for each phone that they produce. Write a linear function that models this situation. Thus use the model to estimate the total costs when 5000 phones are produced in a month.

Solution:

We'll start by defining a variable. We'll let x = the number of phones produced in a month.

Costs are divided into two categories, overhead costs which are fixed and material costs which vary depending on how many phones are produced. We can write a cost function using the given information:

$$C(x) = 69x + 225000$$

To find the total costs when 5000 phones are produced in a month, we'll compute $C(5000)$:

$$C(5000) = 69(5000) + 225000$$
$$= 345000 + 225000$$
$$= 570000$$

The total cost to produce 5000 phones in one month is $570,000.

Example 4: The cost of renting a van is $125 per week plus $0.25 per mile driven. Suppose you rent the van for a week and drive 1275 miles. What will be the total bill?

Solution:

First we'll define a variable. What varies in this situation is the number of miles driven, so we'll let x = the number of miles driven.

Next we'll write a function. The weekly cost is fixed, and we'll multiply $0.25 by the number of miles driven. The function is:

$$C(x) = 125 + 0.25x.$$

Now we can compute $C(1275)$.

$$C(1275) = 125 + 0.25(1275)$$
$$= 125 + 318.75$$
$$= 443.75$$

The total cost is $443.75.

Example 5: Suppose the cost of renting a van from one company is $125 per week plus $0.25 per mile driven. Another company offers the same van for a flat rate of $425 per week. How many miles would you need to drive to make the flat rental rate a better choice?

Solution:

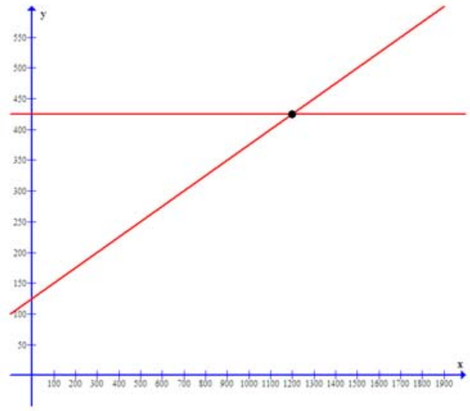
From Example 4, we have a linear function that expresses the cost of renting the van from the first company:

$C(x) = 125 + 0.25x$

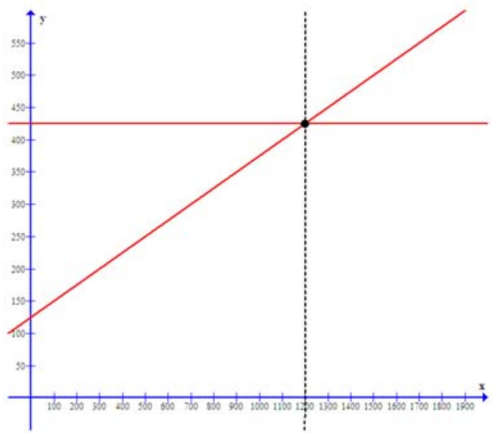
The flat rate charged by the second company can be written as a constant function:

$F(x) = 425$

We can graph these two functions in the coordinate plane:



The point where the two functions intersect is the point where the two costs are the same. We can identify the coordinates of the point by drawing in a vertical line through the point and finding the *x* coordinate. We know that the *y* coordinate is 425.



The point of intersection is (1200, 425). If a customer drives more than 1200 miles, the flat rate is the better deal.