Section 5.4

Solving Quadratic Equations by Factoring

We can use factoring to help up solve quadratic equations. We'll use the zero product property in solving these equations.

The Zero Product Property

Suppose $ab = 0$. Then either $a = 0$ or $b = 0$.

Example 1: Solve by factoring: $x^2 - 4x - 5 = 0$

Solution:

We'll factor the left hand side of the equation and use the zero product property.

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$

Then $x - 5 = 0$ or $x + 1 = 0$

So we have two solutions, $x = 5$ and $x = -1$

Note that we can only use the zero product property when the quadratic equation equals 0. If we do not have 0 on one side of the equal sign, we'll need to do some algebra to transform the equation before factoring.

Example 2: Solve by factoring: $x^2 - x = 6$

Solution:

We'll start by subtracting 6 from both sides of the equation so that we'll have 0 on the right hand side. Then we can factor and use the zero product property.

$$x^2 - x = 6$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \; or \; x + 2 = 0$$
$$x = 3 \; or \; x = -2$$

Our two solutions are $x = 3$ and $x = -2$.

You might be tempted to factor the left side and then set each factor equal to 6. Doing this would give two answers, 0 and 6, neither of which satisfy the equation. The zero product property *requires* that the quadratic equal zero before you start factoring!

Example 3: Solve by factoring: $(x-1)(x+5)=7$

Solution:

We'll start by multiplying the two binomials together. Then, we'll subtract 7 from both sides of the equation. At that point, the quadratic equation will be in the correct form to use the zero product property.

$$\begin{aligned}(x-1)(x+5)&=7\\ x^2+5x-x-5&=7\\ x^2+4x-5&=7\\ x^2+4x-12&=0\end{aligned}$$

Now we can factor the left hand side of the equation and use the zero product property to solve it.

$$\begin{aligned}&x^2+4x-12=0\\ &(x+6)(x-2)=0\\ &x+6=0 \; or \; x-2=0\\ &x=-6 \; or \; x=2\end{aligned}$$

The two answers to the equation are x = -6 and x = 2.

Example 4: Solve by factoring: $9x^2-49=0$

Solution:

We can use the rule for factoring the difference of two squares:

$$\begin{aligned}&9x^2-49=0\\ &(3x+7)(3x-7)=0\\ &3x+7=0 \; or \; 3x-7=0\\ &x=\frac{-7}{3} \; or \; x=\frac{7}{3}\end{aligned}$$

The two answers to the problem are $x=\frac{-7}{3}$ and $x=\frac{7}{3}$.

Example 5: Solve by factoring: $x^2 = 36$

Solution:

We'll start by subtracting 36 from both sides of the equation, so that the right hand side is 0. Then we can factor and use the zero product property to solve the equation:

$$x^2 = 36$$
$$x^2 - 36 = 0$$
$$(x+6)(x-6) = 0$$
$$x + 6 = 0 \text{ or } x - 6 = 0$$
$$x = -6 \text{ or } x = 6$$

The two answer are $x = -6$ and $x = 6$

Example 6: Solve by factoring: $x^2 - 4x + 4 = 0$

Solution:

We'll start by factoring the left hand side of the equation:

$$x^2 - 4x + 4 = 0$$
$$(x-2)(x-2) = 0$$

Now we can use the zero product property to solve. In this case, both of the factors of the left hand side are the same, so we will only have one answer.

$$(x-2)(x-2) = 0$$
$$x - 2 = 0$$
$$x = 2$$

Example 7: Solve by factoring: $3x^2 - x - 4 = 0$

Solution:

First, we need to factor the left hand side of the equation. We'll use factoring by grouping.

We'll need to find *ac*. In this case, it's $3(-4) = -12$. We need to find a pair of factors of -12 whose sum is -1. We can write out all of the factors, as we did earlier, or we can list them mentally until we find the pair that we need. In this case, the pair of factors is 3 and -4.

Now we can rewrite the problem:

$$3x^2 + 3x - 4x - 4 = 0$$

Now we can factor by grouping to solve the problem:

$$3x^2 + 3x - 4x - 4 = 0$$
$$3x(x+1) - 4(x+1) = 0$$

We have a common binomial factor of $x + 1$, which we can factor out:

$$(x+1)(3x-4) = 0$$

Finally, we can use the zero product property to solve the equation:

$$(x+1)(3x-4) = 0$$
$$x+1 = 0 \text{ or } 3x-4 = 0$$
$$x = -1 \text{ or } x = \frac{4}{3}$$

The two solutions to the problem are $x = -1$ and $x = \frac{4}{3}$.