

## Section 5.7

# Solving Problems Using Quadratic Equations

We can use quadratic equations to solve some word problems. As with any word problem, it's helpful to read through the problem until you understand it. Then, draw a picture of the situation, if that's appropriate. Define variables and write an equation. Then you can use one of the methods for solving quadratic equations to solve the equation. Finally, be sure you answer the question that is posed in the problem.

**Example 1:** The sum of two numbers is 21 and their product is 90. Write a quadratic equation and use it to find the two numbers.

Solution:

Start by defining two variables:

Let  $x$  = one of the numbers

Let  $y$  = the other number.

Then we have  $x + y = 21$  and  $xy = 90$ .

Solve  $x + y = 21$  for  $y$  and substitute that quantity into the other equation:

$$x + y = 21$$

$$y = 21 - x$$

So  $x(21 - x) = 90$ . Now we can solve this quadratic equation:

$$x(21 - x) = 90$$

$$21x - x^2 = 90$$

$$-x^2 + 21x - 90 = 0$$

$$x^2 - 21x + 90 = 0$$

$$(x - 15)(x - 6) = 0$$

$$x - 15 = 0 \text{ or } x - 6 = 0$$

$$x = 15 \text{ or } x = 6$$

Using the equation,  $y = 21 - x$ , we can see that when  $x = 15$ ,  $y = 6$  and when  $x$  is 6,  $y$  is 15, so the two numbers are 15 and 6. You can check to see that their sum is 21 and their product is 90.

**Example 2:** A company models its annual profits using the function

$P(x) = x^2 + 20x - 300$ , where  $P$  represents profits and  $x$  gives the number of units sold. One year, their profits were \$167,700. How many units of their product did they sell?

Solution:

We'll let  $x$  = the number of units sold, as indicated in the problem. We want to find the value of  $x$  for which the profit function is 167,700:

$$x^2 + 20x - 300 = 167700$$

We can rewrite the equation, then use the quadratic formula to solve it. This problem may factor, but finding the correct factors could take a lot of time. The formula will probably be quicker.

$$x^2 + 20x - 300 = 167700$$

$$x^2 + 20x - 168000 = 0$$

In this example,  $a = 1$ ,  $b = 20$  and  $c = -168000$ .

$$x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-168000)}}{2(1)}$$

$$x = \frac{-20 \pm \sqrt{400 + 672000}}{2}$$

$$x = \frac{-20 \pm \sqrt{672400}}{2}$$

$$x = \frac{-20 \pm 820}{2}$$

$$x = \frac{-20 + 820}{2} \text{ or } x = \frac{-20 - 820}{2}$$

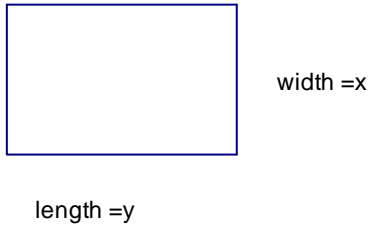
$$x = 400 \text{ or } x = -420$$

The negative answer does not make sense in the context of the problem, so the answer is 400 units.

**Example 3:** The perimeter of a rectangle is 50 cm and its area is 144 square centimeters. Find the dimensions of the rectangle.

Solution:

We'll start by drawing a sketch:



Let  $x$  = width

Let  $y$  = length

Then  $P = 2x + 2y$ . We know that the perimeter is 50 cm, so we can replace  $P$  in this equation with 50:  $50 = 2x + 2y$ . Also,  $A = xy$ . We know that the area is 144 square cm, so we can replace  $A$  in this equation with 144:  $144 = xy$

If  $50 = 2x + 2y$ , we can solve the equation for  $y$  and substitute that quantity into the area formula above:

$$50 = 2x + 2y$$

$$50 - 2x = 2y$$

$$25 - x = y$$

$$144 = x(25 - x)$$

$$144 = 25x - x^2$$

Now we have a quadratic equation which we can solve:

$$144 = 25x - x^2$$

$$x^2 - 25x + 144 = 0$$

$$(x - 9)(x - 16) = 0$$

$$x - 9 = 0 \text{ or } x - 16 = 0$$

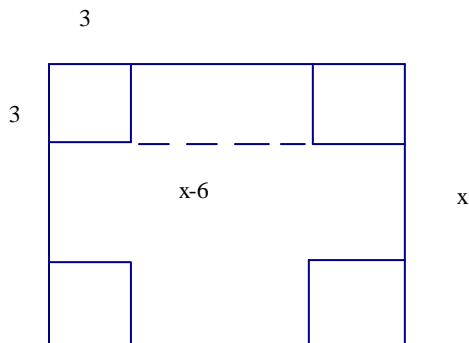
$$x = 9 \text{ or } x = 16$$

Since  $y = 25 - x$ , we know that  $y = 16$  when  $x = 9$  and  $y = 9$  when  $x = 16$ . the dimensions of the rectangle are 16 cm by 9 cm.

**Example 4:** An open top box is made by cutting 3 inch squares from each corner of a square piece of cardboard and then folding up the flaps. The volume of the box is to be 1083 cubic inches. Find the dimensions of the original piece of cardboard.

Solution:

We'll start by drawing a sketch of the situation:



We'll let  $x$  = the measure of each side of the cardboard. The formula for the volume of a box is  $V = lwh$ . In our case, since we are cutting equal squares from each corner, the length and width of the box will each be  $x - 6$ . The height of the box will be the height of the flaps, which is 3 inches.

We can write an equation:

$$V = (x - 6)(x - 6)(3)$$

Since we are given the volume of the box, we can substitute that for  $V$ . We have a quadratic equation:

$$1083 = (x - 6)(x - 6)(3)$$

$$1083 = (x^2 - 6x - 6x + 36)(3)$$

$$1083 = (x^2 - 12x + 36)(3)$$

$$1083 = 3x^2 - 36x + 108$$

We'll rewrite the equation so that the left hand side is 0.

$$0 = 3x^2 - 36x - 975$$

We can divide both sides of the equation by 3. This will make the right hand side easier to factor.

$$0 = 3x^2 - 36x - 975$$

$$0 = x^2 - 12x - 325$$

Now we can factor:

$$0 = (x - 25)(x + 13)$$

$$0 = x - 25 \text{ or } 0 = x + 13$$

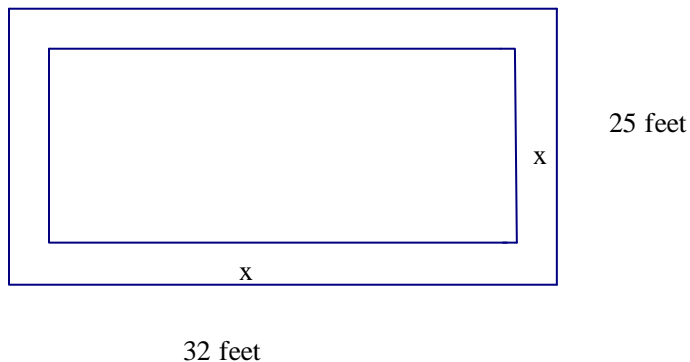
$$25 = x \text{ or } -13 = x$$

The negative answer does not make sense in the context of this problem. So  $x = 25$ .  
The original piece of cardboard measured 25 inches by 25 inches.

**Example 5:** A homeowner wants to install a deck around her swimming pool. The swimming pool is a rectangle and measures 25 feet by 32 feet. The deck is to have the same width all the way around the pool. What should be the width of the deck if she has 310 square feet of decking material?

Solution:

We'll start by drawing a sketch of the situation:



Our approach to this problem will be to find the area of the entire region and then subtract from that the area covered by the swimming pool. What will be left is the area devoted to the deck.

The area of the entire region can be expressed as:

$$A = (32 + 2x)(25 + 2x)$$

$$A = 800 + 64x + 50x + 4x^2$$

$$A = 800 + 114x + 4x^2$$

The area of the swimming pool is  $25 \cdot 32 = 800$

So the area of the deck can be expressed as

$$A = 800 + 114x + 4x^2 - 800$$

$$A = 114x + 4x^2$$

We know that the area of the decking will be limited to the amount of decking material on hand, which is 310 square feet. We can now write the quadratic equation that we need to solve.

$$310 = 114x + 4x^2$$

$$0 = 4x^2 + 114x - 310$$

$$0 = 2x^2 + 57x - 155$$

$$x = \frac{-57 \pm \sqrt{57^2 - 4(2)(-155)}}{2(2)}$$

$$x = \frac{-57 \pm \sqrt{3249 + 1240}}{4}$$

$$x = \frac{-57 \pm \sqrt{4489}}{4}$$

$$x = \frac{-57 \pm 67}{4}$$

$$x = \frac{-57 + 67}{4} \text{ or } x = \frac{-57 - 67}{4}$$

$$x = 2.5 \text{ or } x = -31$$

The negative answer does not make sense in this context, so we know that the width of the deck should be 2.5 feet.