

Section 2.3

Selected Answers

1. a. $3^2 = 9$

d. $-3^2 = (-1)(3)(3) = -9$

b. $3^{-1} = \frac{1}{3}$

e. $-3^{-2} = (-1)\left(\frac{1}{3}\right)^2 = -\frac{1}{9}$

c. $-3^{-1} = -\frac{1}{3}$

3. a. $25^{\frac{1}{2}} = 5$

d. $25^{-2} = \frac{1}{625}$

b. $25^{-1} = \frac{1}{25}$

e. $25^2 = 625$

c. $25^{-\frac{1}{2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$

5. a. $36^{\frac{1}{2}} = 6$

d. $12^{-2} = \frac{1}{144}$

b. $144^{-\frac{1}{2}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$

e. $81^{\frac{3}{4}} = (81^{\frac{1}{4}})^3 = (3)^3 = 27$

c. $125^{\frac{2}{3}} = (125^{\frac{1}{3}})^2 = ((5^3)^{\frac{1}{3}})^2 = 5^2 = 25$

7. a. $\left(\frac{1}{3}\right)^{-2} = (((3^{-1})^{-1})^2) = 3^2 = 9$

b. $(.1)^{-2} = ((10^{-1})^{-1})^2 = 10^2 = 100$

c. $-5^0 = -1$

d. $64^{\frac{2}{3}} = (((4^3)^{\frac{1}{3}})^2) = 4^2 = 16$

$$e. \left(\frac{16}{25}\right)^{-\frac{1}{2}} = \left(\left(\frac{16}{25}\right)^{\frac{1}{2}}\right)^{-1} = \left(\frac{4}{5}\right)^{-1} = \frac{1}{\frac{4}{5}} = 1 \cdot \frac{5}{4} = \frac{5}{4}$$

$$9. \quad a. \left(\frac{1}{5}\right)^{-2} = \left(\left(5^{-1}\right)^{-1}\right)^2 = 5^2 = 25$$

$$b. \left(\frac{1}{49}\right)^{\frac{1}{2}} = \frac{1}{7}$$

$$c. \left(\frac{5}{17}\right)^{-1} = \frac{1}{\frac{5}{17}} = 1 \cdot \frac{17}{5} = \frac{17}{5}$$

$$d. \left(\frac{169}{16}\right)^{-\frac{1}{2}} = \left(\left(\frac{169}{16}\right)^{\frac{1}{2}}\right)^{-1} = \left(\frac{13}{4}\right)^{-1} = \frac{4}{13}$$

$$e. (2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{1}{5}\right)^{-2} = \left(\left(5^{-1}\right)^{-1}\right)^2 = 5^2 = 25$$

$$11. \quad a. 2^5(2^{-3}) = 2^{5-3} = 2^2 = 4$$

$$b. 5^{-1}(5^{\frac{2}{3}})(5^{\frac{4}{3}}) = 5^{-1+\frac{2}{3}+\frac{4}{3}} = 5^{\frac{-3+6}{3}} = 5^{\frac{3}{3}} = 5$$

$$c. -1^0(2^{-1})(4) = -2^{-1}(2^2) = -2$$

$$d. 4^{\frac{1}{2}}(4^{\frac{3}{2}}) = 4^{\frac{1}{2}+\frac{3}{2}} = 4^{\frac{4}{2}} = 4^2 = 16$$

$$e. 12^{\frac{1}{5}}(12^{-\frac{6}{5}}) = 12^{\frac{1}{5}-\frac{6}{5}} = 12^{-\frac{5}{5}} = 12^{-1} = \frac{1}{12}$$

$$13. \quad a. \frac{5^{-2}}{5} = 5^{-2-1} = \frac{1}{5^3} = \frac{1}{125}$$

$$b. \frac{12^0}{12^{-1}} = 12^{0-(-1)} = 12^1 = 12$$

$$\text{c. } \frac{5^{\frac{1}{3}}}{5^{\frac{5}{3}}} = 5^{\frac{1}{3} - \frac{5}{3}} = 5^{\frac{1+5}{3}} = 5^{\frac{6}{3}} = 5^2 = 25$$

$$\text{d. } \frac{2^3}{2^{-2}} = 2^{3-(-2)} = 2^5 = 32$$

$$\text{e. } \frac{3^{\frac{4}{2}}}{3^{\frac{2}{3}}} = 3^{\frac{4}{2} - \frac{2}{3}} = 3^{\frac{6}{3}} = 3^2 = 9$$

$$15. \text{ a. } -(5^2)^{\frac{1}{2}} = -(5^{2 \cdot \frac{1}{2}}) = -5$$

$$\text{b. } (-5^0)^2 = (-1 \cdot 5^0)^2 = (-1)^2 (5^{0 \cdot 2}) = 1$$

$$\text{c. } (5^6)^{\frac{1}{3}} = 5^{6 \cdot \frac{1}{3}} = 5^2 = 25$$

$$\text{d. } (10^{-1})^2 = 10^{-2} = \frac{1}{100} = .01$$

$$\text{e. } (3^3)^{12} = 3^4 = 81$$

$$17. \text{ a. } (48)^{\frac{1}{2}} = (16 \cdot 3)^{\frac{1}{2}} = 16^{\frac{1}{2}} (3^{\frac{1}{2}}) = 4\sqrt{3}$$

$$\text{b. } (3\sqrt{5})^2 = 3^2 (5^{\frac{1}{2}})^2 = 9 \cdot 5 = 45$$

$$\text{c. } 2^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} \cdot 36^{\frac{1}{3}} = (2 \cdot 3 \cdot 6^2)^{\frac{1}{3}} = (6^1 \cdot 6^2)^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6$$

$$\text{d. } (3000)^{\frac{1}{3}} = (3 \cdot 1000)^{\frac{1}{3}} = (3 \cdot 10^3)^{\frac{1}{3}} = 3^{\frac{1}{3}} \cdot 10 = 10\sqrt[3]{3}$$

$$\text{e. } (54)^{\frac{1}{3}} = (3 \cdot 18)^{\frac{1}{3}} = (2 \cdot 3^3)^{\frac{1}{3}} = 3(2)^{\frac{1}{3}} = 3\sqrt[3]{2}$$

19. a. $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

b. $\left(\frac{2\sqrt{3}}{9}\right)^2 = \frac{2^2(3^{\frac{1}{2}})^2}{9^2} = \frac{4 \cdot 3}{9 \cdot 9} = \frac{4 \cdot 3}{3 \cdot 9} = \frac{4}{27}$

c. $\left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ proper-ness!

d. $\left(\frac{121}{169}\right)^{\frac{1}{2}} = \frac{(11^2)^{\frac{1}{2}}}{(13^2)^{\frac{1}{2}}} = \frac{11}{13}$

e. $(.2)^3 = \left(\frac{2}{10}\right)^3 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$

21. a. $\left(\frac{1}{\sqrt{2}}\right)^{-2} = \left(\left(\frac{1}{2^{\frac{1}{2}}}\right)^2\right)^{-1} = \left(\frac{1^2}{(2^{\frac{1}{2}})^2}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$

b. $(125)^{\frac{1}{3}} = \left((5^3)^{\frac{1}{3}}\right)^{-1} = 5^{-1} = \frac{1}{5}$

c. $-103^0 = -1(103)^0 = -1$

d. $(4\sqrt[3]{5})^{\frac{3}{2}} = (4)^{\frac{3}{2}}(\sqrt[3]{5})^{\frac{3}{2}} = (2^2)^{\frac{3}{2}}(5^{\frac{1}{3}})^{\frac{3}{2}} = 2^3(5^{\frac{1}{2}}) = 8\sqrt{5}$

e. $\left(\frac{64}{125}\right)^{\frac{2}{3}} = \left(\left(\frac{4^3}{5^3}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

23. a. $\frac{1}{2}(2)^{-3}(2^3)^2 = 2^{-1-3+6} = 2^2 = 4$

b.
$$\frac{-111^0(5007)^0}{2^{-2}}$$

c.
$$\left(\frac{1}{5}\right)^{\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}} \cdot \frac{5^{\frac{1}{2}}}{5^{\frac{1}{2}}} = \frac{\sqrt{5}}{5} \quad \text{proper-ness!}$$

d.
$$\frac{4^{-1}(16)}{4^3} = 4^{-1+2-3} = 4^{-2} = \frac{1}{16}$$

e.
$$\frac{1005^{\frac{3}{2}}}{1005^{\frac{1}{2}}} = 1005^{\frac{3}{2} - \frac{1}{2}} = 1005$$