2.8 Exercises – problems with asterisks have solutions appended

Give five different representations of the following numbers: as a Rational number, in Scientific Notation (exactly or approximation if you need to round off), factored to primes or showing primes multiplied in a representation, factored but not necessarily to primes, and one of your own choosing.

1. 36

2. 45

3.* 150

4. 225

5. 2,004

6. 16^{-1}

7.* 81^{-1}

8. 1000^{-1}

9. –111

10. -36^{-1}

11.* -27^{-1}

13. -4^{-2}

14. -3^{-2}

15.* -6^{-2}

Convert the following rational numbers to a fraction form.

16. $2.\overline{0125}$

17. $0.\overline{34}$

18. $0.\overline{1234}$

19.* $3.\overline{372}$

20. $-.\overline{113}$

21.* $-.\overline{0145}$

22. $4.022\overline{45}$

23.* $3.224\overline{123}$

24. $-1.03\overline{67}$

25.* $-2.3\overline{16}$

Give an illustration that shows that the following subsets of the Real numbers are not closed under the given operation:

26. Odd natural numbers and division.

27. Rational numbers and division.

28. Whole numbers and subtraction.

29.* Irrational numbers and addition.

30. Integers and division.

Tell which arithmetic property was used in each step of the following worked out problems. (note: some of the steps involve short cuts from the section on exponents, too).

31.

$7(5+3)+7(2)$

$7(5)+7(3)+7(2)$

$7(5)+7(3+2)$

$7(5+5)=7(10)=70$

32.

$5^{\frac{1}{2}}(15^{\frac{1}{2}}-20^{\frac{1}{2}})$

$5^{\frac{1}{2}}\cdot(3\cdot5)^{\frac{1}{2}}-5^{\frac{1}{2}}(4\cdot5)^{\frac{1}{2}}$

$5^{\frac{1}{2}}\cdot3^{\frac{1}{2}}\cdot5^{\frac{1}{2}}-5^{\frac{1}{2}}\cdot4^{\frac{1}{2}}\cdot5^{\frac{1}{2}}$

$5^{\frac{1}{2}}\cdot5^{\frac{1}{2}}\cdot3^{\frac{1}{2}}-5^{\frac{1}{2}}\cdot5^{\frac{1}{2}}\cdot3^{\frac{1}{2}}$

$(5^{\frac{1}{2}}\cdot5^{\frac{1}{2}})\cdot3^{\frac{1}{2}}-(5^{\frac{1}{2}}\cdot5^{\frac{1}{2}})\cdot4^{\frac{1}{2}}$

$5\cdot3^{\frac{1}{2}}-5(2)=5\sqrt{3}-10$

33.*

$5^{-1}x - 2(5^{-1})x$

$x(\frac{1}{5} - \frac{2}{5})$

$-5^{-1}x$

34.

$3(2^{\frac{1}{2}}) + 5(16^{\frac{1}{2}}) - 2(2^{\frac{1}{2}})$

$3(2^{\frac{1}{2}}) + -2(2^{\frac{1}{2}}) + 5(16^{\frac{1}{2}})$

$2^{\frac{1}{2}} + 20$

35.*

$6^{\frac{1}{2}}(3^{\frac{1}{2}} - 3(2^{\frac{1}{2}}))$

$6^{\frac{1}{2}}(3^{\frac{1}{2}}) + 6^{\frac{1}{2}}(-3)(2^{\frac{1}{2}})$

$(6 \cdot 3)^{\frac{1}{2}} + 6^{\frac{1}{2}}(2^{\frac{1}{2}})(-3)$

$(6 \cdot 3)^{\frac{1}{2}} + (6 \cdot 2)^{\frac{1}{2}}(-3)$

$(2 \cdot 3^2)^{\frac{1}{2}} + (3 \cdot 2^2)^{\frac{1}{2}}(-3)$

$(2 \cdot 3^2)^{\frac{1}{2}} + (-3)(3 \cdot 2^2)^{\frac{1}{2}}$

$3(2)^{\frac{1}{2}} + (-3)(2)(3)^{\frac{1}{2}}$

$3\sqrt{2} - 6\sqrt{3}$

Solutions:

3. $150 = \frac{300}{2} = 1.5\times10^{2} = 2\cdot3\cdot5^{2} = 15(10) = 22{,}500^{\frac{1}{2}}$

7. $81^{-1} = \frac{1}{81} = \frac{1}{3^{4}} = 3^{-4} = 9^{-2} = 8.1\times10^{-1}$

11. $-27^{-1} = -\frac{1}{27} = -\frac{1}{3^{3}} = (-3(9))^{-1} = -.\overline{037} \approx -3.7\times10^{-1}$

15. $-6^{-2} = -\frac{1}{36} = -\frac{1}{2^{2}\cdot3^{2}} = -(4\cdot9)^{-1} = .02\overline{7} \approx 2.8\times10^{-2}$

19. $3.\overline{372}$

$x = 3.\overline{372}$

steps 1 and 2

$1000x = 3372.\overline{72}$

$1000x = 3372.\overline{372}$ subtracting the equations

$x = \quad 3.\overline{372}$

$999x = 3369$

$x = \frac{3369}{999}$

21. $-.\overline{0145}$

$x = -.\overline{0145}$

steps 1 and 2

$10000\,x = -145.\overline{0145}$

$$10000x = -145.\overline{0145}$$
$$-\quad x = \quad -.\overline{0145}$$
$$999x = -145$$

subtracting: watch those negatives!

$$x = -\frac{145}{999}$$

23. $3.224\overline{123}$

Get new Equation 1 because of the offset to the repeat bar

$$x = 3.224\overline{123}$$

$$1000x = 3224.\overline{123}$$

Get Equation 2

$$1000000x = 3224123.\overline{123}$$

Now, subtract

$$1000000x = 3224123.\overline{123}$$
$$1000x = \quad 3224.\overline{123}$$
$$999000x = 3220899$$

$$x = \frac{3220899}{999000}$$

25. $-2.3\overline{16}$

Get new equation 1

$$x = -2.3\overline{16}$$

$$10x = -23.\overline{16}$$

Get equation 2 and subtract

$$\begin{array}{rl} & 1000x = -2316.\overline{16} \\ - & \quad 10x = \quad -23.\overline{16} \\ \hline & 990\,x = \ -2296 \end{array}$$

$$x = -\frac{2296}{990}$$

29. Irrational numbers and addition.

$-\pi + \pi = 0$

33.

$5^{-1}x - 2(5^{-1})x$

$(\frac{1}{5} - \frac{2}{5})x$ ”un-distribution” factor out the x, combine the fractions

$-5^{-1}x$

35.

$6^{\frac{1}{2}}(3^{\frac{1}{2}} - 3(2^{\frac{1}{2}}))$

1. $6^{\frac{1}{2}}(3^{\frac{1}{2}}) + 6^{\frac{1}{2}}(-3)(2^{\frac{1}{2}})$

2. $(6\cdot 3)^{\frac{1}{2}} + 6^{\frac{1}{2}}(2^{\frac{1}{2}})(-3)$

3. $(6\cdot 3)^{\frac{1}{2}} + (6\cdot 2)^{\frac{1}{2}}(-3)$

4. $(2\cdot 3^2)^{\frac{1}{2}} + (3\cdot 2^2)^{\frac{1}{2}}(-3)$

5. $(2\cdot 3^2)^{\frac{1}{2}} + (-3)(3\cdot 2^2)^{\frac{1}{2}}$

6. $3(2)^{\frac{1}{2}} + (-3)(2)(3)^{\frac{1}{2}}$

7. $3\sqrt{2} - 6\sqrt{3}$

1. Distribution
2. Associative and Commutative
3. Exponent Powered Product Rule
4. Factoring to Primes
5. Commutative
6. Exponent Powered Product Rule
7. rewriting in traditional notation