

Math 2303  
Section 1.7  
Dimensional Analysis

Unit Fraction –

Rational numbers:  $\frac{1}{2} = \frac{1}{2} * \frac{3}{3} = \frac{3}{6}$ , multiplying by  $1 = \frac{3}{3}$  does NOT change the value of the fraction.

With measurement, a fraction made up of equivalent measurements in different units is considered multiplying by 1.

Examples of Unit Fractions:

$$\frac{1kg}{1000g}, \frac{100cm}{1m}, \frac{1in}{2.54cm}, \frac{5ml}{1tsp}$$

Multiplying a measurement in one unit by a unit fraction so that the units cancel has the effect of transforming the numerical value to the equivalent measurement in the other unit.

Example

$$2in = 2in \left( \frac{2.54cm}{1in} \right) = 5.08cm$$

$$\frac{2in}{1} \cdot \frac{2.54cm}{1in}$$

$$0.001km = 0.001km \left( \frac{1000m}{1km} \right) = 1m$$

$$45ml = 45ml \left( \frac{1tsp}{5ml} \right) = 9tsp$$

How to construct the appropriate unit fraction or fractions:

Units of Length, Mass and Volume

**Example 1:** convert 409 cm to inches:

$$\frac{409cm}{1} \cdot \frac{1in}{2.54cm} = 161in$$

**Example 2:** Convert 0.44 km to miles

$$\frac{0.44km}{1} \cdot \frac{1miles}{1.609km} = 0.273miles$$

**Example 3:** Convert 155 lb to kilograms

$$\frac{155 \cancel{\text{lb}}}{1} \cdot \frac{.4536 \text{ kg}}{1 \cancel{\text{lb}}} = 70.3 \text{ kg}$$

**Example 4:** Convert 3.5 grams to ounces

$$\frac{3.5 \text{ g}}{1} \cdot \frac{1 \text{ oz}}{28.35 \text{ g}} = .123 \text{ oz}$$

**Example 5:** Convert 20 gallons to liters

$$\frac{20 \text{ gallons}}{1} \cdot \frac{3.785 \text{ liter}}{1 \text{ gallon}} = 75.7 \text{ liters}$$

**Example 6:** Convert 12 ml to fluid ounces

$$\frac{12 \text{ ml}}{1} \cdot \frac{1 \text{ fl oz.}}{29.57 \text{ ml}} = .406 \text{ fluid oz.}$$

Sometimes, the units you want to convert between do not have a direct equivalence in the chart you are using. In that case you need to construct a chain of units to get from one to the other.

**Example 7:** Convert 4320 cm to yards

$$\frac{4320 \text{ cm}}{1} \cdot \frac{1 \text{ m}}{100 \text{ cm}} \cdot \frac{1 \text{ yard}}{.9144 \text{ m}} = \frac{4320}{100 \cdot .9144} = 47.2 \text{ yards}$$

**Example 8:** Convert 20 fluid ounces to liters

$$\frac{20 \text{ oz}}{1} \cdot \frac{29.57 \text{ ml}}{1 \text{ oz}} \cdot \frac{1 \text{ liter}}{1000 \text{ ml}} = \frac{20 \cdot 29.57}{1000} \text{ l} = .5914 \text{ liters}$$

**Example 9:** Convert 1 cubic foot to liters

$$\frac{1 \text{ ft}^3}{1} \cdot \frac{.02832 \text{ m}^3}{1 \text{ ft}^3} \cdot \frac{1 \text{ kL}}{1 \text{ m}^3} \cdot \frac{1000 \text{ liters}}{1 \text{ kL}} = 28.32 \text{ liters}$$

Converting Area and Volume using dimensions;

**Example 10:** Find the area, in square meters, of a field that is 25 meters wide and 0.12 kilometers long.

$$\frac{0.12 \text{ km}}{1} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 120 \text{ m}$$

$$\text{Area} = 25 \text{ m} \times 120 \text{ m} = 3000 \text{ m}^2$$

**Example 11:** A box is 12 inches wide by 1.5 feet long by 1.5 feet tall.

- Find the volume of the box in cubic inches.
- Find the volume of the box in cubic meters.
- Find the area of the box in cubic decimeters.

$$1.5 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 18 \text{ in}$$

$$\text{Volume} \quad 12 \text{ in} \cdot 18 \text{ in} \cdot 18 \text{ in} = 3888 \text{ in}^3$$

$$\text{Volume in } \text{ft}^3 \quad 1 \text{ ft} \cdot 1.5 \text{ ft} \cdot 1.5 \text{ ft}$$

$$\frac{2.25 \text{ ft}^3}{1} \cdot \frac{0.02832 \text{ m}^3}{1 \text{ ft}^3} = 0.0637 \text{ m}^3$$

**Example 12:** Velocity: Convert 55 miles per hour to kilometers per hour.

$$\frac{55 \text{ miles}}{1 \text{ hr}} \cdot \frac{1.609 \text{ km}}{1 \text{ mile}}$$

$$= \frac{88.5 \text{ km}}{1 \text{ hr}}$$

$$= 88.5 \text{ km/hr}$$

$$\frac{0.0637 \text{ m}^3}{1} \cdot \left(\frac{10 \text{ dm}}{1 \text{ m}}\right)^3$$

$$= \frac{0.0637 \text{ m}^3}{1} \cdot \frac{1000 \text{ dm}^3}{1 \text{ m}^3} = 63.7 \text{ dm}^3$$

$$1 \text{ mile} = 5280 \text{ ft}$$

$$1 \text{ hr} = 3600 \text{ sec}$$

**Example 13:** Convert 55 miles per hour to feet per second.

$$\frac{55 \text{ miles}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ miles}} \cdot \frac{1 \text{ hr}}{3600 \text{ sec}}$$
$$= \frac{55 \cdot 5280}{3600} \text{ ft/sec} = 80.7 \text{ ft/sec}$$

**Example 14:** Price: Convert \$2.25 per linear foot to price per meter.

$$\frac{\$ 2.25}{1 \text{ ft}} \cdot \frac{1 \text{ ft}}{30.48 \text{ cm}} \cdot \frac{100 \text{ cm}}{1 \text{ m}}$$
$$1 \text{ ft} = 30.48 \text{ cm}$$
$$= \$ 7.38 / \text{meter}$$