

Math 2303
 Concepts in Algebra
 Section 2.2
 Integers

If there is \$100 in my checking account and I write a check for \$150, how much is in my account?

If it is $0^{\circ}C$ outside and a strong cold front drops the temperature another 20 degrees, what is the temperature outside?

If I describe walking forward 6 steps as + 6, how do I describe backing up 3 steps?

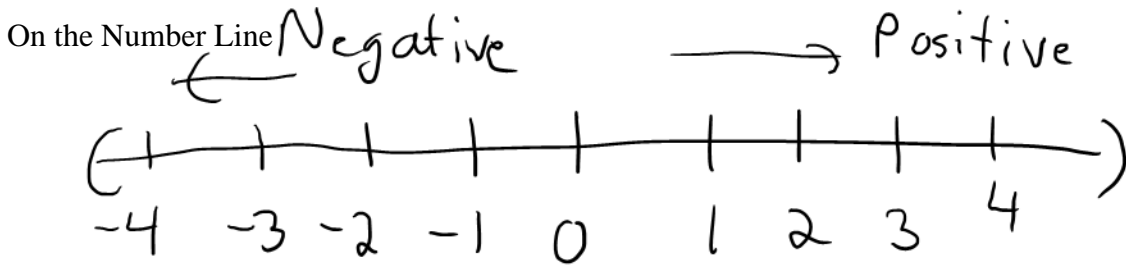
None of these questions have natural number answers. We need symbols for more numbers to describe these very real situations.

Adding more numbers:
 FIRST, ADD ZERO to our list of numbers

Whole Numbers = $\{0, 1, 2, 3, 4, 5, 6, \dots\}$

But, we still need more so create **NEGATIVE NUMBERS** to be the opposite of **POSITIVE COUNTING NUMBERS**. The set with negative numbers, zero and natural numbers together is called the **INTEGERS**. The symbol for the integers is \mathbb{Z} .

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$



\leftarrow Smaller Bigger \rightarrow
 -4 is smaller than -1



$2 + 3 = 5$
 right 2 $2 + (-3)$ left 3 = -1



Absolute Value - the absolute value of a number is its distance from 0, which is the "number part" without the negative sign.

$$|3| = 3$$

$$|-2| = 2$$

If there is an expression inside the absolute value bars, simplify, then take off the negative sign -

$$|-6 - 3(-1)| = |-6 + 3| = |-3| = 3$$

Rules for arithmetic with integers

ADDITION AND SUBTRACTION - If you struggle with these, I HIGHLY recommend that you read section 2.2 in the textbook. The stairs illustration is very helpful.

Example: Simplify $5 + (-77) = -72$

$$|5| = 5$$

$$|77| = 77$$

$$5 + (-77)$$

$$77 - 5 = 72$$

$$-14 + 9$$

$$= -5$$

$$\begin{array}{r} 14 \\ -9 \\ \hline 5 \end{array}$$

$$19 - (-5)$$

$$= 19 + 5 = 24$$

$$-152 + 99 - 12$$

$$\begin{array}{r} 99 \\ -12 \\ \hline 87 \end{array}$$

$$\begin{array}{r} 152 \\ -87 \\ \hline 65 \end{array}$$

$$-152 + 87 = -65$$

$$-18 + (-5)$$

$$= -23$$

add numbers
stick on negative

$$1 - 4 = 1 + (-4)$$

Subtraction and Negative
are equivalent

$$10 + -4 = 10 - 4$$

$$3 - 5 + 7 - 8 = 3 + (-5) + 7 + (-8)$$

MULTIPLICATION AND DIVISION

Like Signs -

$$(-5)(-20) = +100$$

$$8 * 4 = 32$$

$$-3 * -5 = 15$$

$$-2 * -6 = 12$$

$$-55 \div -5 = 11$$

$$\frac{-120}{-8} = 15$$

$$84 / (5-9) = \frac{84}{5-9} = \frac{84}{-4} = -21$$

$$2 \times 3 = \text{How many things in 2 groups of 3}$$

$$= 3 + 3$$

2 terms

Opposite Signs -

$$-8 * 6 = -48$$

$$11 * -6 = -66$$

$$-6 + -6 + -6 + -6 + -6 + -6 + -6 + -6 + -6 + -6 + -6$$

11 terms

$$8(-4) = -32$$

$$2 * -12 = -24$$

$$-60 / 12 = -5$$

$$1000 \div -8 = -125$$

$$(-3)(-4)(-5) = -60$$

odd # negative sign
= neg answer

$$\frac{72}{-3} = -24$$

$$= \frac{-72}{3} = -\frac{72}{3}$$

$$\frac{-108}{9} = \frac{108}{-9} = -\frac{108}{9}$$

$$= -12$$

Operations with Zero

Addition -

$$3 + 0 = 3$$

$$0 + x = x$$

Subtraction -

$$4 - 0 = 4$$

$$0 - 4 = -4 = 0 + -4 = -4$$

Multiplication -

$$6 * 0 = 0$$

Nonzero Number
 $3 \times \frac{1}{12,502}$ answer
 is NOT
 zero

Interpretation - How many things in six sets, each with 0 things in it?

OR - How many total things in zero sets of 6?

$$6 \times 0 = 0$$

Division -

$$\frac{0}{x}$$

$$0 \div 6 = 0$$

take 0 things split into 6 sets
 there are 0 things in each set

$$\frac{x}{0}$$

to have a solution this must have ONE answer, every time I calculate it - so lets see

$$\frac{0}{0} = ?$$

$\frac{0}{0} = \text{something}$ then $0 \times \text{something} = 0$ SOMETHING
 NOT
 UNIQUE

$$\frac{3}{0} = ?$$

$\frac{3}{0} = \text{whatever}$
 $0 \times \text{whatever} = 3$
 NOT POSSIBLE

YOU CANNOT DIVIDE BY ZERO!!!!