

Math 2303
 Concepts in Algebra
 Section 2.5
 Irrational Numbers

Rational Numbers - $\frac{a}{b}$ with a and b integers and b not equal to 0.

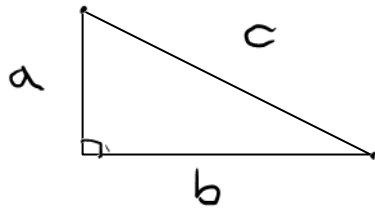
With rational numbers we can describe parts of the whole -

$$\frac{3}{40}$$

Divide 1 whole into 40 parts
 keep 3

Do we have all the numbers we need to describe physical distances?

What about -



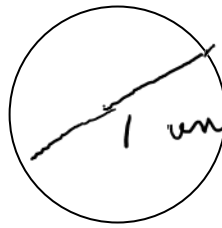
$$a^2 + b^2 = c^2$$

$$a = 1 \quad b = 1$$

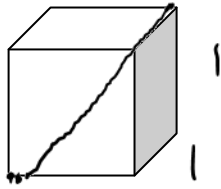
$$1^2 + 1^2 = c^2$$

$$c^2 = 2$$

$$c = \sqrt{2}$$



1 unit diameter



diagonal

$\sqrt{3}$ in length
 not rational number

$$\text{Circumference} = \pi d$$

$$= \pi \cdot 1 = \pi$$

about 3.14

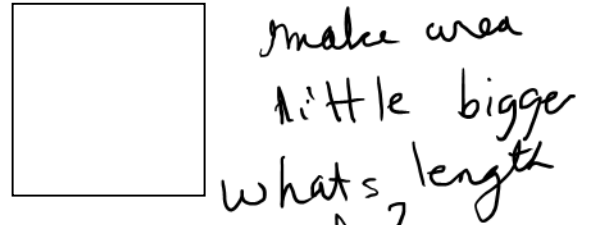
All of these lengths cannot be described precisely as ratios of integers. Thus they are NOT RATIONAL NUMBERS.

Definition: An irrational number is a real number that cannot be described precisely as a ratio of integers.

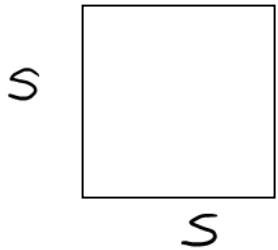
Examples: $\sqrt{2}, \sqrt{5}, \pi$

Square Roots that
 dont simplify exactly
 is irrational

Squares:



Area 10 in^2
what length of side = $\sqrt{10} \text{ in}$ of side?



Area = side \cdot side = side²
side = 3 in
Area 9 in²

Area = Length * Width = Side * Side

Example 1: Given a square with the following area, find the length of a side. Is the length of the side a rational number or an irrational number?

(a). Area = 9 ft^2

Side = 3 ft $\sqrt{9} = 3$

(b). Area = $\frac{121}{49} \text{ cm}^2$

Side $\sqrt{\frac{121}{49}} = \frac{\sqrt{121}}{\sqrt{49}} = \frac{11}{7} \text{ cm}$

(c). Area = $6m^2$

Side $\sqrt{6}$ m

(d). Area = $0.4in^2$

Side $\sqrt{0.4}$ in

$\sqrt{0.4} = 0.632455\dots$
irrational

Example 2: Given a square with sides of the following lengths, find the area of the square.

(a). Side = $9in$

Area = $9 \times 9 = 81 in^2$

(b). Side = $\sqrt{7}$ in

Area $\sqrt{7} \cdot \sqrt{7} = 7 in^2$

$\sqrt{x} \cdot \sqrt{x} = x$

$\sqrt{2} \cdot \sqrt{2} = 2$

$\sqrt{10} \cdot \sqrt{10} = 10$

(c). Side = $\frac{2}{3}$ in

Area $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$

Circles

$$C = \pi d$$

$$\frac{C}{d} = \pi$$

Approximating pi -
3.14

Textbook 2.5 100 decimal
places for π

$\frac{22}{7}$

Exactly pi -

π

calculator $\boxed{\pi}$ key

Find the circumference of a circle with diameter 5 inches

(a). Using 3.14 for pi

(b). Using $\frac{22}{7}$ for pi

(c). Using the pi key on a calculator -

(d). Exactly -