

Math 2303  
Sections 2.6 and 2.7  
Number Theory

Multiplication Notation for Natural Numbers

If  $\mathbf{a} \times \mathbf{b} = \mathbf{c}$ , then a and b are called factors of c, and c is called a multiple of a and also a multiple of b.

Division Notation for Natural Numbers

If  $\frac{\mathbf{c}}{\mathbf{a}} = \mathbf{b}$  and a,b and c are natural numbers (i.e. the division works out evenly), then a and b are called divisors of c.

Since  $\frac{\mathbf{c}}{\mathbf{a}} = \mathbf{b}$  if and only if  $\mathbf{a} \times \mathbf{b} = \mathbf{c}$ , divisors of c and factors of c are the same things!

The factors of 24 are –

The divisors of 15 are –

The divisors of 7 are -

A prime number is a natural number greater than 1 that has exactly two factors (divisors), itself and 1. A natural number greater than 1 that is not prime is called a composite number because it is “composed” of the product of two nontrivial factors.

First few primes 2,3,5,7,11,13,17,19..

How to test if a number is prime.

Greek Method – Sieve of Eratoschenes

- find primes less than or equal to a given natural number.
- list the numbers from 1 to the given number.
- Cross off 1.
- Circle the first prime, 2, and then cross off all multiples of 2.
- Circle the next prime, 3, and then cross off all multiples of 3.
- Continue in this fashion. The next number which is not crossed off after eliminating the multiples of the last prime is prime.
- When you circle the first prime ,p, such that  $p \cdot p$  is greater than the last number in your list, you are done.

Example 1: Determine the primes less than or equal to 75.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75					

### Divisibility Test Method

This method works for determining if a number up to 120 is prime. The rules also determine if a large number is divisible by the natural number given.

Test the number for divisibility by each of the first 10 natural numbers.

Natural Number	Test
2	If the last digit is even
3	If the sum of the digits is divisible by 3
4	If the last 2 digits are divisible by 4
5	If the last digit is 5 or 0
6	If the number is divisible by 2 and 3
7	Multiply the last digit by 2 and subtract this from the number formed by the rest of the digits, if this is divisible by 7, so is the original number
8	If the last 3 digits of a number are divisible by 8, so is the number.
9	The sum of the digits is divisible by 9
10	The last digit of the number is a 0

Example 2: Is the number 97 prime? Use the divisibility tests to determine if it is divisible by any number 10 or less.

### Divisibility by Primes Test

To test if a number is prime, we really only need to test and see if it is divisible by smaller primes. Why?

Check all primes up to the prime  $p$  such that  $p \cdot p$  is bigger than the number we are testing.

Example 3: Test and see if 221 is prime.

#### Primes in Modern Mathematics

We can trace the beginnings of the concept of prime numbers back to the Greek mathematicians.

Even with modern technology like the computer, some questions the Greeks asked about prime numbers are still not fully answered.

Some examples of questions we still do not know the answers to:

Twin Prime Conjecture

#### Factors and Multiples of Composite Numbers

##### Prime Factorization

The Fundamental Theorem of Arithmetic

Every natural number can be expressed as a unique (up to the order of the factors) product of prime numbers.

Example:  **$36 = 2 \times 2 \times 3 \times 3$**

Example 4: Find the prime factorization of 60.

Example 5: Find the prime factorization of 1236.

Example 6: Find the prime factorization of 23310.

Example 7: Find the prime factorization of 19

Greatest Common Divisor (GCD)

Also known as the greatest common factor (GCF)

The greatest common divisor (GCD) of a set of natural numbers is the largest natural number that divides (without remainder) every number in the set.

Two natural numbers are called relatively prime if their GCD is 1.

Finding the GCD by the prime factorization method.  
Example 8: Find the GCD of the numbers 48 and 80.

Example 9: Find the GCD of the numbers 56 and 72.

Example 10: Find the GCD of the numbers 24, 60 and 252.

#### Least Common Multiple

The least common multiple of a set of natural numbers is the smallest natural number that is divisible by (without remainder) each number in the set.

Example: The least common multiple of 6 and 4 is 12.

Finding the LCM by the prime factorization method.  
Example 11: Find the LCM of the numbers 24 and 42.

Example 12: Find the LCM of the numbers 150 and 525.

Example 13: Find the LCM of the numbers 540, 144 and 198.

Example 14: List all the factors of 900.  
Since 2 and 4 are factors of 900, is 8 a factor?  
Since 5 and 3 are factors of 900, is 15 a factor?