

Math 2303
Section 3.5
Formulas

Volume and Surface Area Example from last time:

A box is going to be made out of solid oak. The material costs \$3.25 per square foot.

There are two options being considered for the dimensions of the box.

Option 1: The box is 2 feet long, 1.5 foot wide and 3 feet high.

Option 2: The box is a cube with sides of length 2 ft.

Which box has the bigger volume?

Option 1
 9 ft^3

Option 2
 $2^3 = 8 \text{ ft}^3$

Which box will cost the most to make (material is surface area)?

$$SA = 2wl + 2wh + 2lh$$

Option 1 $SA = 2(1.5)(2) + 2 \cdot 1.5 \cdot 3$
 $+ 2 \cdot 2 \cdot 3$

$$= 6 + 9 + 12 = 27 \text{ ft}^2$$

Option 2 $SA = 6s^2$ cubic box
 $= 6 \cdot 2^2 = 24 \text{ ft}^2$

A water tank is in the shape of a cylinder with radius 10 m and height 20 ft. Find the volume of the cylinder in cubic meters and in cubic feet.

Cylinder: $A = 2\pi r^2 + 2\pi rh$
 $V = \pi r^2 h$

$$r = 10 \text{ m} \quad 1 \text{ ft} = 0.3 \text{ m}$$

$$r = \frac{10 \text{ m}}{1} \cdot \frac{1 \text{ ft}}{0.3 \text{ m}} = \frac{10}{0.3} \text{ ft}$$

$$= 33.33 \text{ ft}$$

$$h = \frac{20 \text{ ft}}{1} \cdot \frac{0.3 \text{ m}}{1 \text{ ft}} = 6 \text{ m}$$

Meters

$$V = \pi r^2 h = \pi (10)^2 \cdot 6 = 600\pi \text{ m}^3$$

Volume
feet²

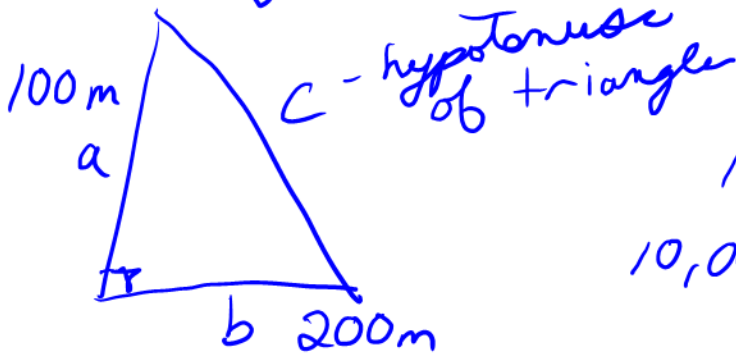
$$V = \pi (33.33)^2 \cdot 20$$

multiply
round
to 2 decimal

A field is in the shape of a right triangle with legs of length 100 m and 200 m. How long is the hypotenuse of this triangle?

Pythagorean Theorem

$$a^2 + b^2 = c^2$$



$$100^2 + 200^2 = c^2$$

$$10,000 + 40,000 = c^2$$

$$c^2 = 50,000$$

$$c = \sqrt{50,000}$$

$$= 223.6$$

The population of a county was 60,000 at the start of a study. The population grew exponentially with a growth constant of 0.021. How many people were in the county 10 years after the study began?

$$P = 60,000 e^{.021 \times 10}$$

$$= 74,021$$

$$P = P_0 e^{kt}$$

P_0 - initial population
 k - growth constant
 t - time in years

The amount of pollution in a pond decays exponentially with a decay constant of 0.00323 when time is measured in days. If the initial amount of pollutant in the pond was 34.5 mg, how much was left in the pond 14 days later?

$$Q = Q_0 e^{-kt}$$

Q - amount at end
 Q_0 - amount started with
 t - time

$$Q_0 = 34.5 \text{ mg} \quad t = 14$$

$$k = .00323$$

$$Q = 34.5 \cdot e^{-.00323 \cdot 14} \text{ mg}$$

$$= 32.97 \text{ mg}$$

Financial Formulas
Simple Interest

$$\begin{aligned} & \text{7.5\% out of 100} \\ & \text{7.5\%} = \frac{7.5}{100} \\ & = .075 \end{aligned}$$

$$I = p \cdot r \cdot t$$

p = amount invested
 r = interest rate as a decimal
 t = time in years

Future Value with simple interest

Start with P dollars
earn simple interest I
end investment period you have A
dollar

$$A = P + I = P + PRt = P(1 + Rt)$$

Present Value with simple interest

Monthly Payment

Suppose you are buying a car and there are two options you are considering for financing your purchase. Option 1 is to take the dealer financing at the special low interest rate of 3.9% compounded monthly. Option 2 is to take the dealer offered rebate of \$2000 and finance through the bank at 8.85% per year compounded monthly. Which is the better deal? How much will you save over the life of the loan?

$$\text{Formula: } R = \frac{Pi(1+i)^n}{(1+i)^n - 1}$$

Option 1: Dealer Financing

Borrow 14,000 = P

Monthly interest rate $i = \text{APR}/12 = 0.039/12 = 0.00325$ (3.9% as a decimal equals 3.9/100)

$$M = \frac{14,000(0.00325)(1 + 0.00325)^{48}}{(1 + 0.00325)^{48} - 1} = 315.48$$

For dealer financing, you will pay \$315.48 per month for 48 months.

Option 2 – Rebate and Bank Financing

Borrow 12,000 = P

Monthly interest rate $i = \text{APR}/12 = 0.0885/12 = 0.007375$ (note, do NOT round this number!!)

$$M = \frac{12,000(0.007375)(1 + 0.007375)^{48}}{(1 + 0.007375)^{48} - 1} = 297.77$$

For the rebate option, you will pay \$297.77 per month for 48 months.

I would chose OPTION 2, the rebate.
How much will I save over the course of the loan?

I make 48 payments.

Option 1 total cost = monthly payment * 48
 $315.48 * 48 = \$15143.04$

Option 2 total cost = monthly payment * 48
 $297.77 * 48 = 14292.96$

Total Savings = $15,143.04 - 14,292.96 = \850.08