Math 2303
Sections 5.4
Solving Quadratic Equations by Factoring

Remember that solving an equation is finding a value or values of the variable such that the quantities on both sides of the equals sign are the same. The most common techniques of solving equations involve algebraically manipulating the equation so that it becomes

Expression = 0

Why is this easier to solve? Because of the ZERO PRODUCT PROPERTY

If ab = 0 then a = 0 or b = 0

The simplest techniques for solving quadratics, or even other polynomials, is to factor them into linear factors and then set the factors equal to zero.

Why factor?

$(x-10)(x+3) = 0$

When is $x^2 - 7x - 30 = 0$? Without factoring, this is not so easy to see, but

When is $x - 10 = 0$ or when is $x + 3 = 0$ are easy questions to answer.

Basic factoring rule
YOU MUST have Expression = 0 to solve by factoring, so move everything to one side before trying to factor.
Factoring Patterns
1. Square minus square
Example 1: Solve $x^2 - 49 = 0$

$x(x-2) = 3$

$x^2 - 2x = 3$

$-3 \quad -3$

$x^2 - 2x - 3 = 0$

$(x+7)(x-7) = 0$

$a^2 - b^2 = (a+b)(a-b) = 0$

Example 2: Solve: $4x^2 - 25 = 0$

$2^2x^2 - 5^2 = 0$

$(2x)^2 - 5^2 = 0$

$(2x-5)(2x+5) = 0$

$2x - 5 = 0$ or $2x + 5 = 0$

$+5 \quad +5$ $\qquad$ $-5 \quad -5$

$2x = 5$ $\qquad$ $x = 5/2$ $\qquad$ $2x = -5$ $\qquad$ $x = -5/2$

This is square MINUS square

Example 3: Solve $x^2 + 9 = 0$

$x = 0$

$0^2 + 9 = 9$

9 is smallest $x^2 + 9$ can be

$(-3)^2 + 9 = 0$

$9 + 9 = 18$ NOT

$3^2 + 9 = 18$

NO REAL SOLUTION

Pattern 2: Quadratics with the leading coefficient (the coefficient of the x-squared term) equal to 1.

Example 4: Solve $x^2 - 2x - 63 = 0$

$(x + 7)(x - 9) = 0$

$$\begin{aligned} x + 7 &= 0 \\ -7 \quad & -7 \\ \hline x &= -7 \end{aligned}$$

$$\begin{aligned} x - 9 &= 0 \\ +9 \quad & +9 \\ \hline x &= 9 \end{aligned}$$

-63

↑ negative sign says one factor is + other is −

$(x + 7)(x - 9) = x^2 - 9x + 7x - 63$

Example 5: Solve $x^2 + x = 12$

$$\begin{aligned} -12 \quad & -12 \\ \hline x^2 + x - 12 &= 0 \end{aligned}$$

$(x + 4)(x - 3) = 0$

$$\begin{aligned} x + 4 &= 0 \\ -4 \quad & -4 \\ \hline x &= -4 \end{aligned}$$

$$\begin{aligned} x - 3 &= 0 \\ +3 \quad & +3 \\ \hline x &= 3 \end{aligned}$$

Example 6: Solve: $x^2 + 36 = 12x$

$$\begin{aligned} -12x \quad & -12x \\ \hline x^2 - 12x + 36 &= 0 \end{aligned}$$

$(x - 6)(x - 6) = 0$

$$\begin{aligned} x - 6 &= 0 \\ +6 \quad & +6 \\ \hline \end{aligned}$$

$\boxed{x = 6}$

Pattern 3: Techniques when the x-squared term has a coefficient other than one:
Technique 1: Guess
Factor and solve $2x^2+5x-3=0$

Factors of 2 are 1, 2
Factors of 3 are 1, 3

$(x+3)(2x-1)=0$

$(x-3)(2x+1)$ NOT right

$x+3=0$
$-3 \quad -3$
$\boxed{x=-3}$

$2x-1=0$
$+1 \quad +1$
$2x=1$
$\boxed{x=1/2}$

Technique 2: Factor by Grouping
Solve $4x^2-20x+25=0$

$ax^2+bx+c=0$

$a=4 \quad b=-20 \quad c=25$

$a\cdot c=4\times 25=100$

Two numbers that multiply $=100$ add up to $b=-20$

$-10\times -10=100$

$-10+-10=-20$

$4x^2-10x-10x+25=0$

$2x(2x-5)-5(2x-5)=0=(2x-5)(2x-5)$

$=(2x-5)^2=0$

$2x-5=0$

$x=5/2$

Solve: $3x^2-13x-10=0$

Technique 3: Common Factor Removal

Solve: $4x^2 - 4x - 15 = 0$

$ac = -60$

$b = -4$

Two numbers

Mult $= -60$

Add $= -4$

2×-30 $\quad$ 10×-6

3×-20 $\quad$ 12×-5

6×-10

Prelim Factor

$\left(\frac{4x}{2} + \frac{6}{2}\right)\left(\frac{4x}{2} - \frac{10}{2}\right)$

Factored $(2x+3)(2x-5)$

Solve: $10x^2 + 9x - 9 = 0$

$a = 10 \quad b = 9 \quad c = -9$

$ac = -90$

Two #s $\quad$ mult $= -90$ $\quad$ add $= 9$

-3×30

-6×15 *

Prelim $\left(\frac{10x}{2} \frac{-6}{2}\right)\left(\frac{10x}{5} \frac{+15}{5}\right)$

Factored $(5x-3)(2x+3) = 0$

$$5x - 3 = 0 \qquad\qquad 2x + 3 = 0$$

$$+3 \quad +3 \qquad\qquad -3 \quad -3$$

$$\frac{5x}{5} = \frac{3}{5} \qquad\qquad \frac{\cancel{2}x}{\cancel{2}} = \frac{-3}{2}$$

$$x = 3/5 \qquad\qquad x = -3/2$$