Solving Equations by Factoring

Definition: The **zero-product property** says that if a and b are numbers such that $ab = 0$, then $a = 0$ or $b = 0$ (or both).

Definition: A **quadratic equation** is an equation that can be written as $ax^2 + bx + c = 0$, where a, b, and c are numbers and $a \neq 0$.

Solving Quadratic Equations
To solve a quadratic equation, we must find all possible values for x that make $ax^2 + bx + c = 0$.

Factoring is usually a helpful way to solve quadratic equations. To use factoring, move all nonzero terms to one side of the equal sign so that the other side is zero. Then use the zero-product property.

Examples:

1. Solve the equation $x^2 - 5x - 24 = 0$.
   Only the left-hand side (LHS) of the equation has nonzero terms, so no movement of terms is necessary. Factor the LHS and use the zero-product property (ZPP):

   $x^2 - 5x - 24 = 0$
   Factor: $(x + 3)(x - 8) = 0$
   ZPP: $x + 3 = 0$ or $x - 8 = 0$
   Solve for x: $x = -3$ or $x = 8$

2. Solve $2x^2 + 18x - 72 = 0$ for x.
   All nonzero terms are on the LHS, so no movement of terms is needed.

   $2x^2 + 18x - 72$
   Factor: $2(x + 12)(x - 3) = 0$
   ZPP: $x + 12 = 0$ or $x - 3 = 0$ (Note: $2 \neq 0$, so we don't write it)
   Solve for x: $x = -12$ or $x = 3$

3. Solve the equation $6x^2 - 27x = -12$.
   This equation has nonzero terms on both sides of the equal sign. To make it possible to solve, move the –12 to the LHS by adding 12 to both sides:

   $6x^2 - 27x + 12 = 0$
   Factor: $3(2x - 1)(x - 4) = 0$
   ZPP: $2x - 1 = 0$ or $x - 4 = 0$ (Note: $3 \neq 0$)
   Solve for x: $x = ½$ or $x = 4$

4. Solve $16x^2 = 1$.

Subtract 1 from both sides: $16x^2 - 1 = 0$

Factor: $(4x - 1)(4x + 1) = 0$ (Note: This is a 'special' polynomial.)

ZPP: $4x - 1 = 0$ or $4x + 1 = 0$

Solve for x: $x = \frac{1}{4}$ or $x = -\frac{1}{4}$

5. Solve: $-12x^2 = -17x + 6$

This quadratic equation has nonzero terms on both sides of the equal sign, so we must move all the terms to one side. Add $17x - 6$ to both sides and solve.

$-12x^2 + 17x - 6 = 0$

Factor: $-1(4x - 3)(3x - 2) = 0$ (Note: $-1 \neq 0$)

ZPP: $4x - 3 = 0$ or $3x - 2 = 0$

Solve for x: $x = \frac{3}{4}$ or $x = \frac{2}{3}$

6. Solve for x: $x(x - 2) = -1$

Make one side equal zero: $x(x - 2) + 1 = 0$

Distribute: $x^2 - 2x + 1 = 0$

Factor: $(x - 1)^2 = 0$ (Note: This is a 'special' polynomial.)

ZPP: $x - 1 = 0$ (Note: No need to write $x - 1 = 0$ twice.)

Solve for x: $x = 1$

Solving Other Polynomial Equations

Solving other polynomial equations is done just like the quadratic equations: Set one side of the equation to zero, factor, use the zero-product property, and solve for x.

Examples:

1. Solve $4x^3 + 16x^2 + 15x = 0$.

This polynomial is already equal to zero, so all we need do is factor, use the ZPP, and solve for x.

$4x^3 + 16x^2 + 15x = 0$

Factor: $x(2x + 5)(2x + 3) = 0$

ZPP: $x = 0$ or $2x + 5 = 0$ or $2x + 3 = 0$

Solve for x: $x = 0$ or $x = -\frac{5}{2}$ or $x = -\frac{3}{2}$

2. Solve $x^3 = 2x^2 + 99$.

Get LHS equal to zero: $x^3 - 2x^2 - 99 = 0$

Factor: $x(x - 11)(x + 9) = 0$

ZPP: $x = 0$ or $x - 11 = 0$ or $x + 9 = 0$

Solve for x: $x = 0$ or $x = 11$ or $x = -9$

3. Solve $7x^3 - 14x^2 = 0$.

Set one side equal to zero:	Already done.
Factor:	$7x^2(x - 2) = 0$
ZPP:	$7x^2 = 0$ or $x - 2 = 0$
Solve for x:	$x = 0$ or $x = 2$

4. Solve $x^3 + 18 = 2x^2 + 9x$ by factoring.

Set one side equal to zero:	$x^3 + 18 - 2x^2 - 9x = 0$
Rewrite:	$x^3 - 2x^2 - 9x + 18 = 0$
Factor:	$(x^2 - 9)(x - 2) = 0$
	$(x + 3)(x - 3)(x - 2) = 0$
ZPP:	$x + 3 = 0$ or $x - 3 = 0$ or $x - 2 = 0$
Solve for x:	$x = -3$ or $x = 3$ or $x = 2$

5. Solve $30x^3 - 3x^2 - 9x = 0$.

Set one side equal to zero:	Already done.
Factor:	$3x(5x - 3)(2x + 1) = 0$
ZPP:	$3x = 0$ or $5x - 3 = 0$ or $2x + 1 = 0$
Solve for x:	$x = 0$ or $x = {}^3/_5$ or $x = {}^-\!/_2$

6. Solve $2(x - 1)^2 = 2x(x^2 - 20) + (8x^2 + 2)$.

Set one side equal to zero:	$2(x - 1)^2 - 2x(x^2 - 20) - (8x^2 + 2) = 0$
Simplify:	$2(x^2 - 2x + 1) - 2x(x^2 - 20) - (8x^2 + 2) = 0$
	$2x^2 - 4x + 2 - 2x^3 + 40x - 8x^2 - 2 = 0$
Rewrite:	$-2x^3 - 6x^2 + 36x = 0$
Factor:	$-2x(x + 6)(x - 3) = 0$
ZPP:	$-2x = 0$ or $x + 6 = 0$ or $x - 3 = 0$
Solve for x:	$x = 0$ or $x = -6$ or $x = 3$

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How to solve an equation:

1) Factor the expression.
2) Set each factor equal to 0.
3) Solve each simpler equation.

Solve the following equation by factoring.

Example 1: $x^2 - 49 = 0$

Example 2: $9x^2 - 64 = 0$

Example 3: $x^2 - 4x = 0$

Example 4: $2x^2 - 5x + 3 = 0$

Example 5: $3x^2 + 10x = 8$

Example 6: $-24x^3 - 72x^2 - 48x = 0$

Example 7: $x^3 - 2x^2 - 4x + 8 = 0$

Example 8: $6x^2 + 13x - 15$

Example 9: $16x^3 - 25x$

Example 10: $30x^3 - 3x^2 - 9x$

Example 11: $x^4 + 3x^2 - 4$

Example 12: $4x^3 - 52x^2 - 3x + 39 = 0$