Chapter 1

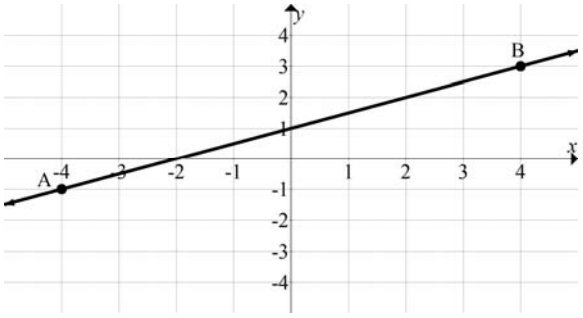
Sections 1.1, 1.2 and 1.3
Linear Equations, Graphs of Lines, Distance and Midpoint Formulas

A linear equation can be written in the:

1. **Point-Slope Form**
   $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line.
2. **Slope-Intercept Form**
   $y = mx + b$, where m is the slope of the line and $b$is the y-intercept of the line.
3. **General Form**
   $\mathrm{Ax} + \mathrm{By} + \mathrm{C} = 0$, where A and B are not both equal to 0.

Example 1: Write the following equation in slope-intercept form and identify the slope and y-intercept $2x - 3y - 5 = 0$.

The **graph** of a linear equation is a straight line like:



The **slope** of a line measures the steepness of a line.

The Slope Formula
The slope of a line that passes through two points (x_1, y_1) and (x_2, y_2) on the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 2: Find the slope of the line containing the points A and B in the picture above.

Example 3: Find the slope of the line containing the following pair of points:

a. $(-3, 1)$ and $(-3, -2)$

b. $(1, 2)$ and $(3, 2)$

Note:

- Lines with positive slope rise to the right.
- Line with negative slope fall to the right.
- Lines with slope equal to 0 are horizontal lines.
- Lines with undefined slope are vertical lines.

Example 4: Write an equation of the line that satisfies the given conditions.

a. $m = -3$ and the line passes through $(-2, 1)$.

Recall: $y - y_1 = m(x - x_1)$

b. line passes through $(-6, 10)$ and $(-2, 2)$.

Recall: $m = \frac{y_2 - y_1}{x_2 - x_1}$ and $y - y_1 = m(x - x_1)$

Let us say that we know want to determine whether a given point is on the graph of a linear equation, without graphing.

We can simply take the point of interest, plug it into the linear equation, and determine if it is satisfied or not. If so, then the point is on the graph of the line; if not, then the point is not on the graph of the line.

Example 5: Determine whether $(-2, -10)$ is on the graph of $y = -3x + 4$.

Example 6: Determine whether $(4, -5)$ is on the graph of $4x - 5y = 41$.

Parallel and Perpendicular Lines
Two lines are parallel if and only if their slopes are the same.

Two lines are perpendicular if and only if their slopes are negative reciprocals of each other.

Example 7: Determine if the following pairs of lines are parallel, perpendicular or neither.

a. $3x - 2y = 5$
   $-9x + 6y = 7$

b. $-3x + y = -5$
   $x + 3y = 7$

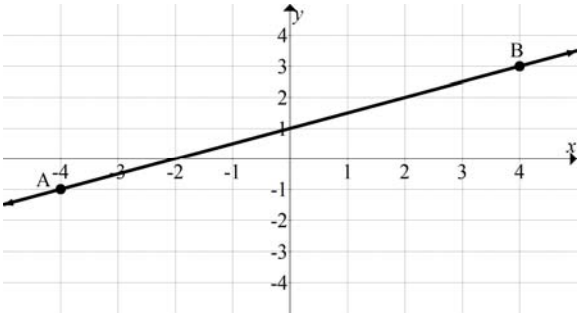
Intercepts of Graphs

An ***x*-intercept** of a graph is the *x*-coordinate of a point where the graph intersects the *x*-axis. To find the *x*-intercept(s) of a graph set $y = 0$ in the equation and solve for x.

A ***y*-intercept** of a graph is the *y*-coordinate of a point where the graph intersects the *y*-axis. To find the *y*-intercept of a graph set $x = 0$ in the equation and solve for y.

Example 8: Given the following graph, determine the *x*- and *y*-intercepts.

x-intercept *y*-intercept



Example 9: Determine the *x*- and *y*-intercepts of the following equation.
$3x - 8y - 12 = 0$

x-intercept *y*-intercept

The last topics of interest are distance and midpoint of a line segment.

The Distance Formula
For any two points (x_1, y_1) and (x_2, y_2), the distance between them is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula
The midpoint of the line segment joining the two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 10: Given the points $(-3, 1)$ and $(1, -4)$,

a. find the distance between them.

b. find the midpoint between them.