Section 2.3: Quadratic Equations

Solving by Factoring

A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$ where *a, b,* and *c* are real numbers with $a \neq 0$.

To solve a quadratic equation by factoring, rewrite the equation, if necessary, so that one side is equal to 0 and use the **Zero-Product Property**:

$ab = 0$ if and only if a = 0 or b = 0.

Example 1: Solve the following equations by factoring.

a. $-3x^2 + 21x = 0$

b. $x^2 - 3x = 18$

c. $5x^2 - 15x + 10 = 0$

d. $2x^2 + 15x - 8 = 0$

e. $6x^2 - x - 12 = 0$

Solving by Square Root Method

Example 2: Solve the following equations by using the square root method.

a. $x^2 - 100 = 0$

b. $9x^2 = 16$

c. $(x-1)^2 = 49$

d. $(x+3)^2 = 12$

Solving by Completing the Square

Given $x^2 + bx + c = 0$

1. Rewrite the equation as $x^2 + bx = -c$
   (Notice that the leading coefficient is positive 1, if it's not then you will have to divide both sides of the equation by the leading coefficient.) and make the left – hand side a perfect square.
2. Make the left-hand side a perfect square by adding $\left(\frac{b}{2}\right)^2$ to both sides

   (to balance the equation)
3. Factor the left-hand side.
4. Use the square root property to solve.

Example 3: Find all real solutions of the following equations by completing the square.

a. $x^2 - 6x - 11 = 0$

b. $x(x+2) = 2$

c. $2x^2 + 16x + 8 = 0$

d. $-3x^2 + 12x + 9 = 0$

Solving by the Quadratic Formula

The solutions of the equation $ax^2 + bx + c = 0$, where $a \neq 0$,
can be found by using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 4: Find all real solutions for $6x^2 - 6x + 1 = 0$ by using the quadratic formula.

Note: The **discriminant** of the equation $ax^2 + bx + c = 0 \, (a \neq 0)$ is given by $D = b^2 - 4ac$.

If D > 0, then the equation $ax^2 + bx + c = 0$ has two distinct real solutions.
If D = 0, then the equation $ax^2 + bx + c = 0$ has exactly one real solution.
If D < 0, then the equation $ax^2 + bx + c = 0$ has no real solution (The roots of the equation are complex numbers and appear as complex conjugate pairs.)

Example 5: Determine the number of real solutions for: $3x^2 + 2x + 2 = 0$