Section 2.4: An Introduction to Complex Numbers

In this section, you'll learn an introduction to complex numbers. Where a complex number has the form of $a + bi$. You will learn to add, subtract, multiply and divide these numbers

Complex Numbers

Definition: A **complex number** is a number that can be written in the form $a + bi$, where a is called the **real part** and bi is called the **imaginary part**. The a and b are real numbers and $i = \sqrt{-1}$

Here are several properties of complex numbers:

Addition of Complex Numbers:
$(a + bi) + (c + di) = (a + c) + (b + d)i$
Add the real parts together and add the imaginary parts together.

Subtraction of Complex Numbers
$(a + bi) - (c + di) = (a - c) + (b - d)i$
Subtract the real parts and subtract the imaginary parts.

Multiplication of Complex Numbers:
Multiply in the same manner as multiplying binomials and remember that $i^2 = -1$

Example 1: Simplify each.

a. $\sqrt{-16}$

b. $\sqrt{-40}$

Example 2: Simplify each of the following and write the answer in form a + b*i*.

a. $(5 + 4i) + (2 - i)$

b. $(-6 - 3i) - (-2 + 2i)$

c. $-i(-3 + 6i)$

d. $(-1 - i)(2 + 5i)$

e. $\sqrt{-16} \cdot \sqrt{-36} + \sqrt{-9} + 10$

Next, you'll need to be able to find various powers of *i*. You'll need to know these 4 powers:

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = i^2 * i = -1 * i = -i$
$i^4 = i^2 * i^2 = -1 * -1 = 1$

For other powers of *i*, divide the exponent by 4 and find the remainder. Your answer will be *i* raised to the remainder power. If the remainder is zero, your answer will be i^4 or 1.

Example 3: Simplify each.

a. i^{15}

b. i^{72}

c. i^{42}

d. i^{313}

Division of Complex Numbers

The **complex conjugate** of the complex number $a + bi$ is the complex number $a - bi$.

To simplify the quotient $\frac{a+bi}{c+di}$ multiply both the numerator and denominator by the complex conjugate of the denominator.

Example 4: Simplify each of the following and write the answer in form a + b*i*.

a.

$$\frac{5}{2-3i}$$

b.

$$\frac{-1-i}{i}$$

c.

$$\frac{5+4i}{4+i}$$

Complex Roots of Quadratic Equations

Using complex numbers, we can now find all solutions to quadratic equations. We can use any of the techniques from the previous section to solve, but usually, we will just take the square root of both sides of the equation, complete the square or use the quadratic formula.

Example 5: Find all complex solutions of the following equations. Express your answer in form a + b*i*.

a. $x^2 + 100 = 0$

b. $49x^2 + 36 = 0$

c. $x^2 - 6x = -13$

d. $4x^2 + 8x + 9 = 0$