Section 2.6: Linear Inequalities

An **inequality in the variable *x*** is linear if each term is a constant or a multiple of *x*.

You'll solve 4 different types of linear inequalities, involving these four symbols:

$<$ less than (the quantity to the left is less than the quantity to the right)
$\leq$ less than or equal to (the quantity to the left is less than or equal to the quantity to the right)
$>$ greater than (the quantity to the left is greater than the quantity to the right)
$\geq$ greater than or equal to (the quantity to the left is greater than or equal to the quantity to the right)

To solve an inequality containing a variable, find all values of the variable that make the inequality true. In solving linear inequalities, isolate the variable on one side of the inequality symbol by using the following rules.

1. If $A < B$ then $A + C < B + C$.
2. If $A < B$ then $A - C < B - C$.
3. Let $C > 0$. If $A < B$ then $AC < BC$.
4. Let $C < 0$. If $A < B$ then $AC > BC$.

Example 1: Solve each of the following inequalities.
a. $-3 \leq 2x + 1$

b. $2(7 - 4x) \geq -13 + 8x$

Next, you'll need to be able to work with interval notation. An interval is a set of real numbers. It can be a line segment, a ray or the entire number line. If it is a line segment, it can include one or both endpoints. If it is a ray it may or may not include the endpoint. We note intervals using brackets, parentheses or a combination.

Example 2: Write each of these inequalities using interval notation.

a. $x < 3$

b. $4 < x \leq 7$

Example 3: Solve each inequality. Graph each solution on the real number line. Write your solutions using interval notation.

a $\frac{5}{6} - \frac{1}{3}x \leq \frac{1}{2}(x+5)$

b $\frac{-3(-x-1)}{8} < \frac{7}{3}$

You can also solve some compound inequalities. All of the same rules apply to these problems

Example 4: Solve each inequality. Write your solutions using interval notation. Graph each solution on the real number line.

a. $-2 \leq x + 5 < 7$

b. $35 < 5x - 5(x - 7)/2 \leq 70$

Sometimes linear inequalities may have no solution or infinitely many solutions.
Let's look at a couple of problems.

Example 5: Solve each of the following inequalities, if possible.

a. $2(x+3) \leq 5x - 3x + 8$

b. $x + 5 + 3x < 4(x + 1)$