Math 1310
Section 3.4: Transforming Functions

 There are two types of transformations:
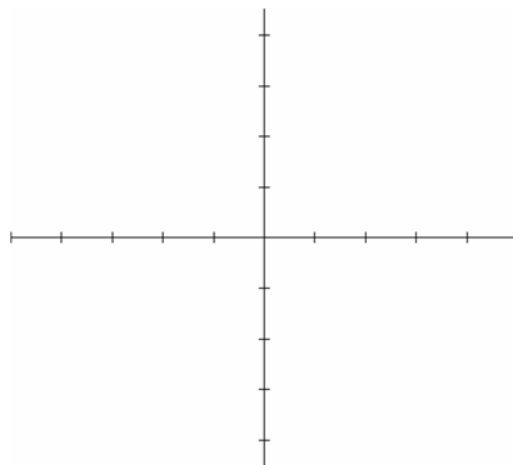
- Translation
- Reflections

We'll start with **translations**. To **translate** a graph means to shift it horizontally, vertically or both.
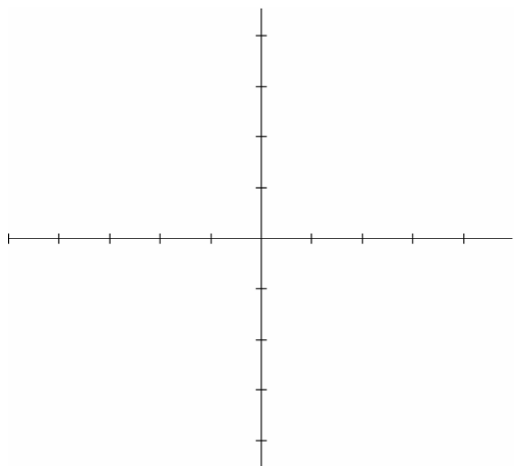
Vertical shifting:

To graph $y = f(x) + c$, $c > 0$, start with the graph of $f(x)$ and shift it upward c units.
To graph $y = f(x) - c$, $c > 0$, start with the graph of $f(x)$ and shift it downward c units.
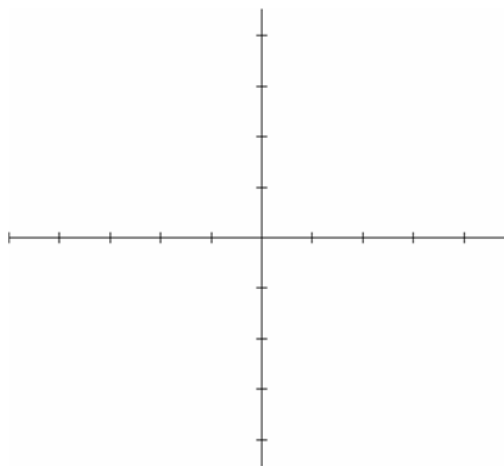
Example 1: Sketch $f(x) = x^2$

$f(x) = x^2 + 2$ $f(x) = x^2 - 2$
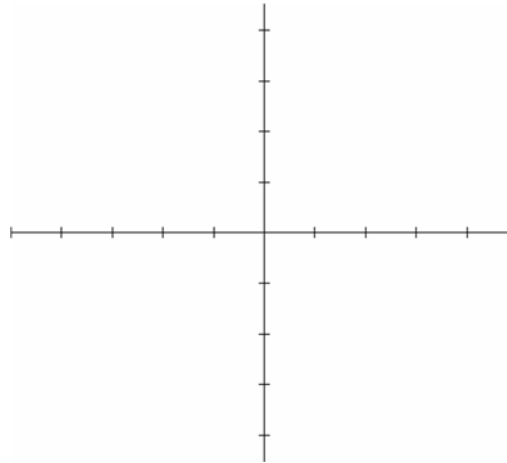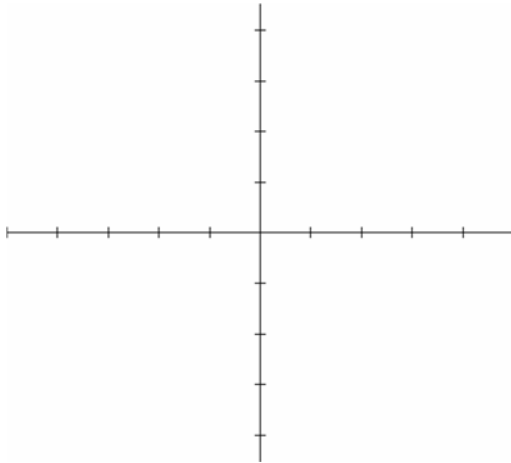
Horizontal shifting:

To graph $y = f(x + c)$, $c > 0$, start with the graph of $f(x)$ and shift it left c units.

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To graph $y = f(x - c)$, $c > 0$, start with the graph of $f(x)$ and shift it right c units.

Example 2: Sketch the following graphs

$f(x) = (x - 3)^2$ $f(x) = (x + 2)^2$

We can also reflect a function. A reflection of a function is its mirror image about the x axis or the y axis.
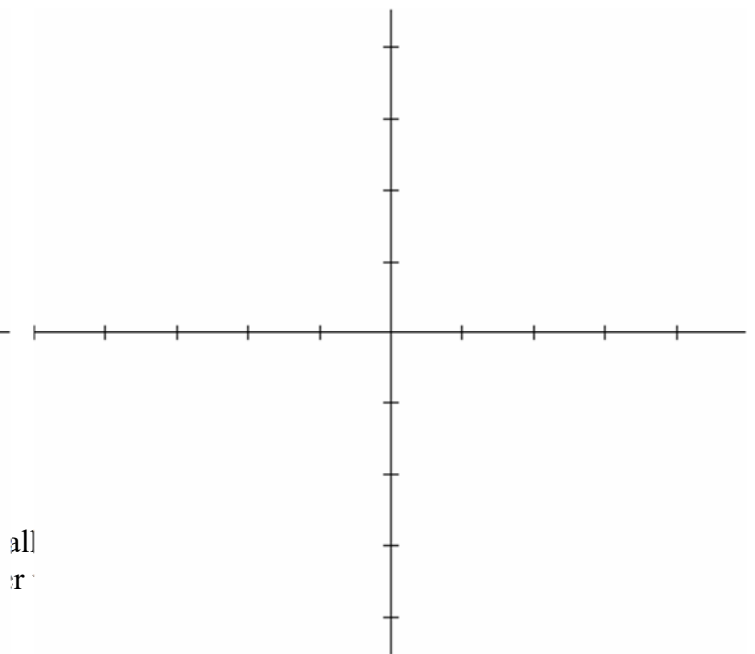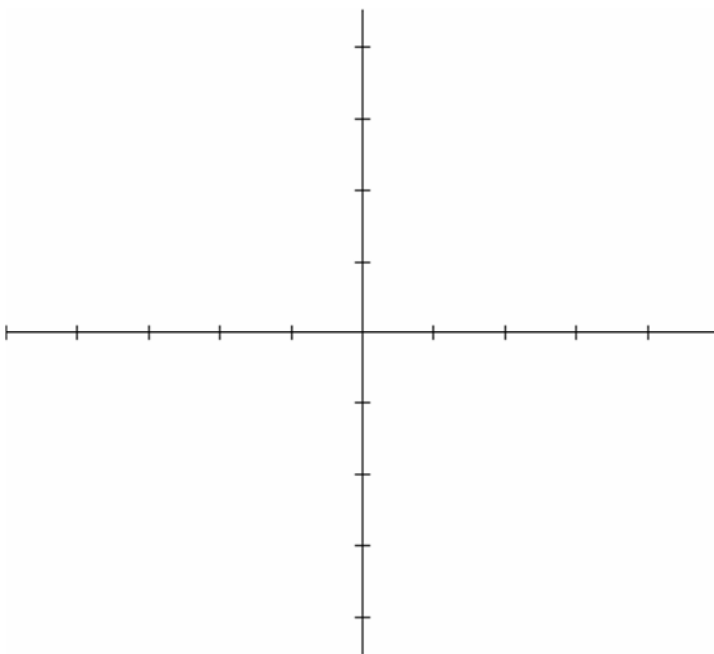
To graph $-f(x)$, reflect the graph of $f(x)$ about the x axis.
To graph $f(-x)$, reflect the graph of $f(x)$ about the y axis.

Example 3: Sketch the following graphs

$f(x) = -\sqrt{x}$ $f(x) = \sqrt{-x}$
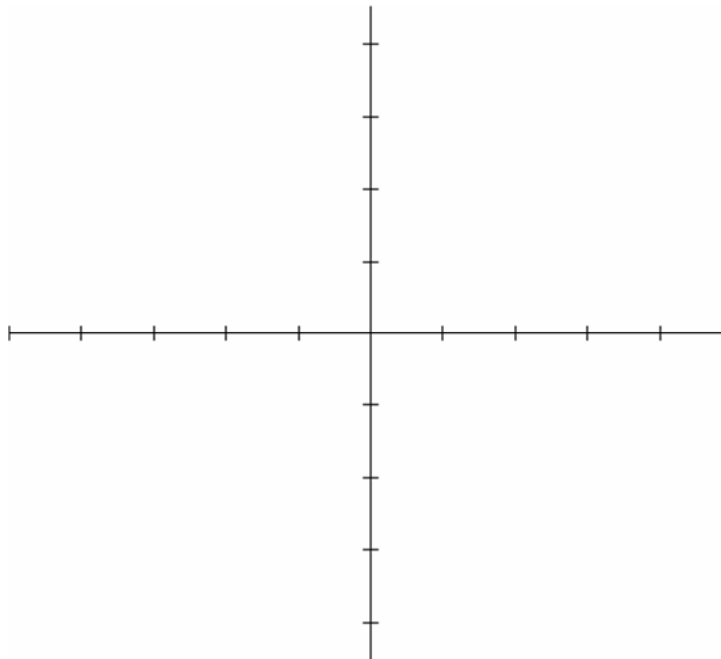
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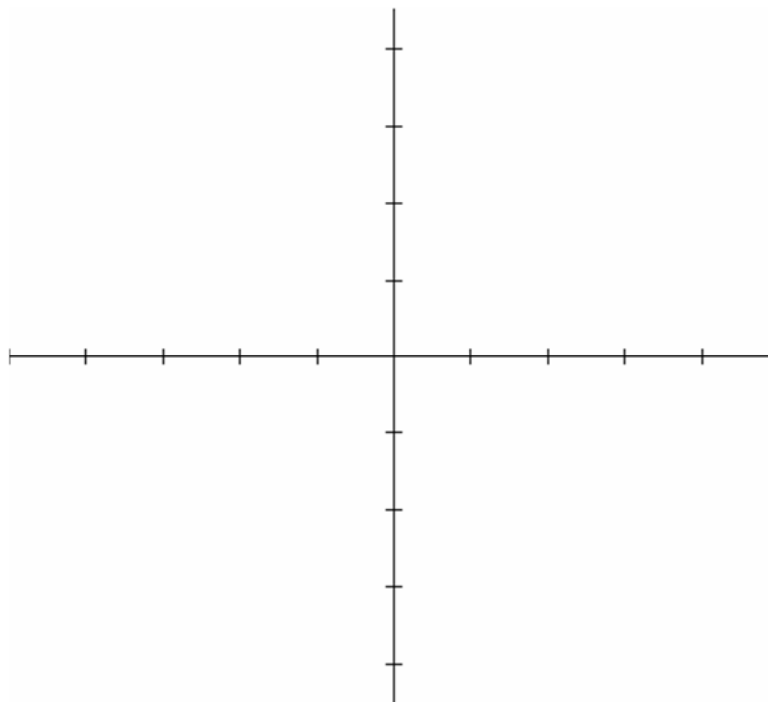
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To graph $y = af(x)$, $a > 1$, stretch the graph of $f(x)$ by a factor of a.

To graph $y = af(x)$, $0 < a < 1$, shrink the graph of $f(x)$ by a factor of $\frac{1}{a}$.

Example 4: Sketch a graph of $f(x) = 3\sqrt[3]{x}$.

Example 5: Sketch a graph of $f(x) = \frac{1}{3}x^2$.

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Recommended order for transforming functions:

1. Vertically stretch or shrink the function.
2. Reflect the function about the x axis.
3. Translate the function vertically and/or horizontally.
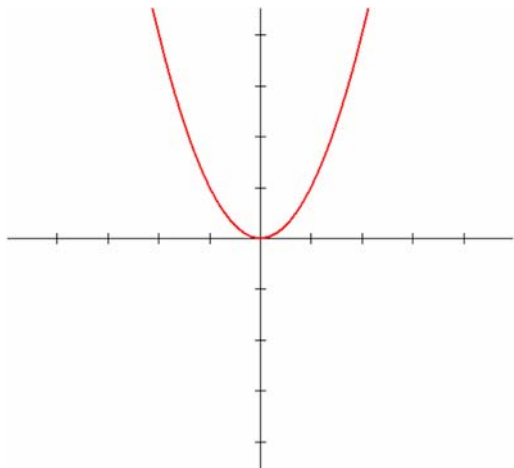4. Reflect the function about the y axis.

**Note, not all of these transformations will be presented in each problem. This is not the only order that works, but this order will get the job done with the fewest mistakes. Memorize this order!!!
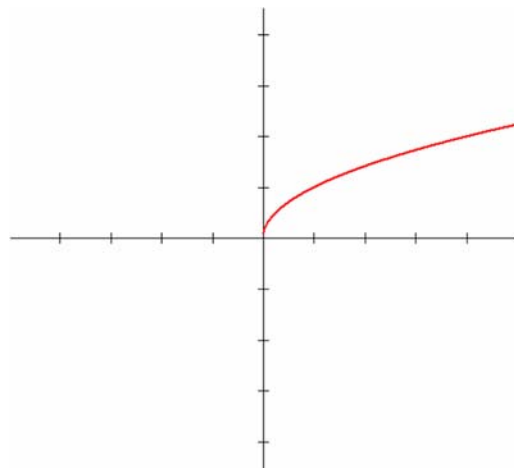
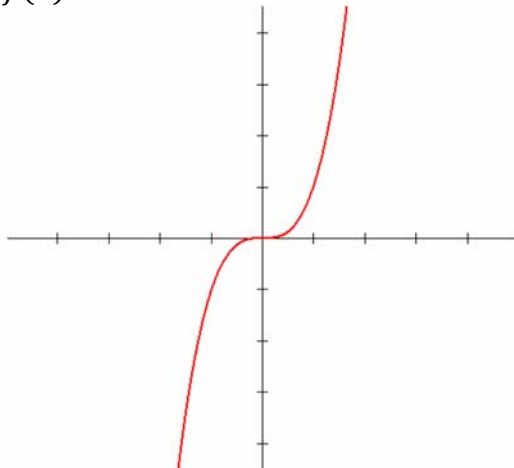**Example 6:** Sketch the graph of $f(x) = -3(x - 2)^2 - 1$.
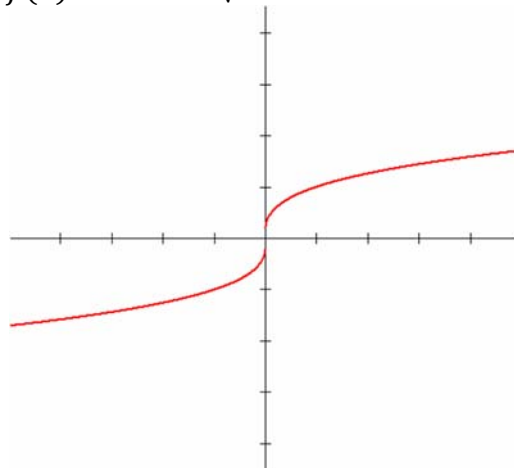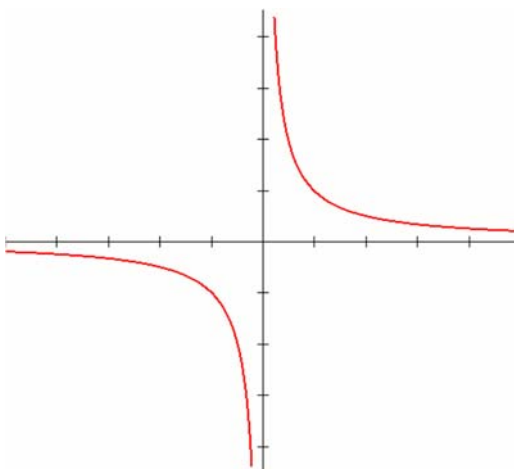

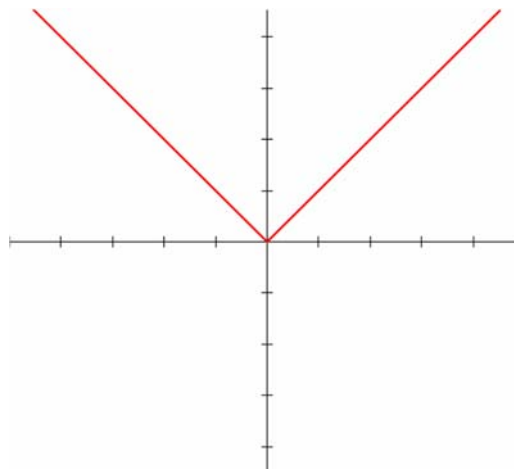
4
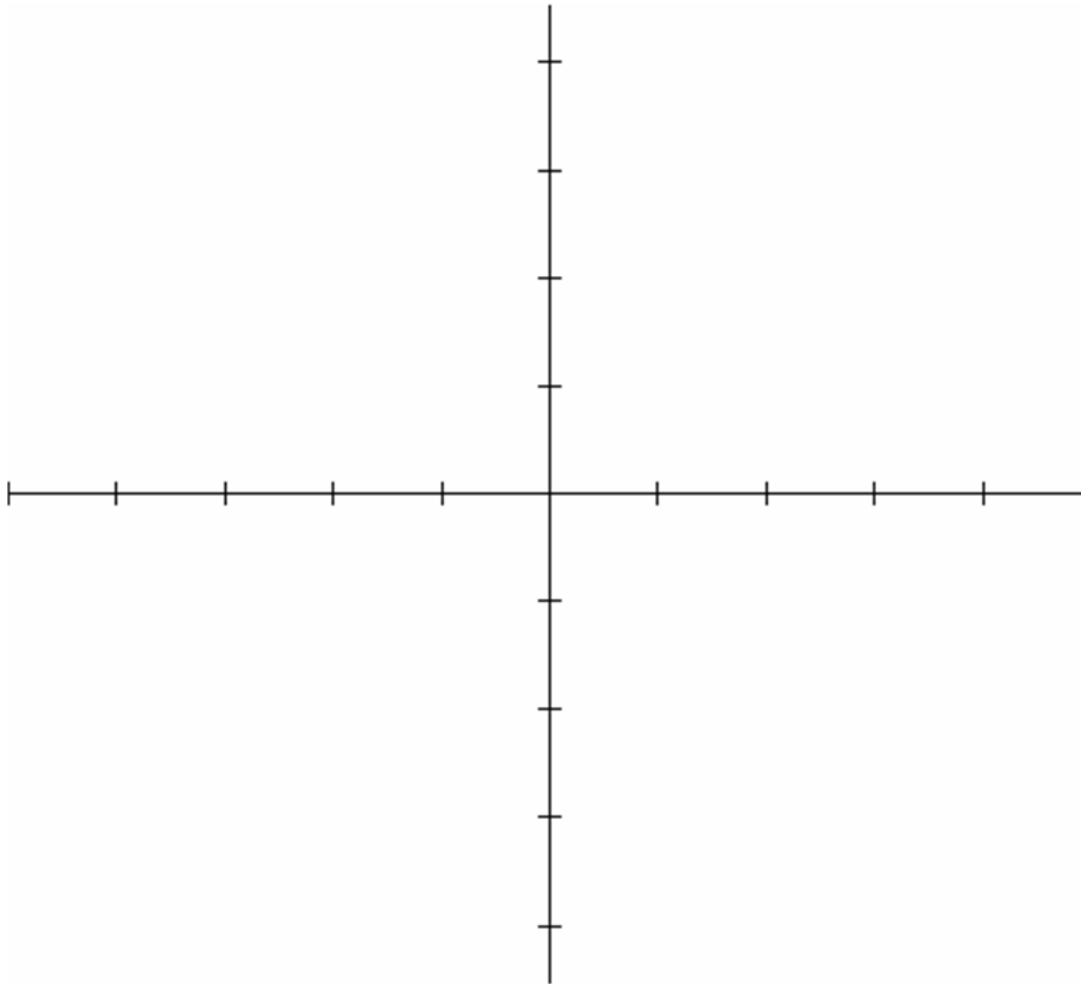
Catalog of Functions

$f(x) = x^2$

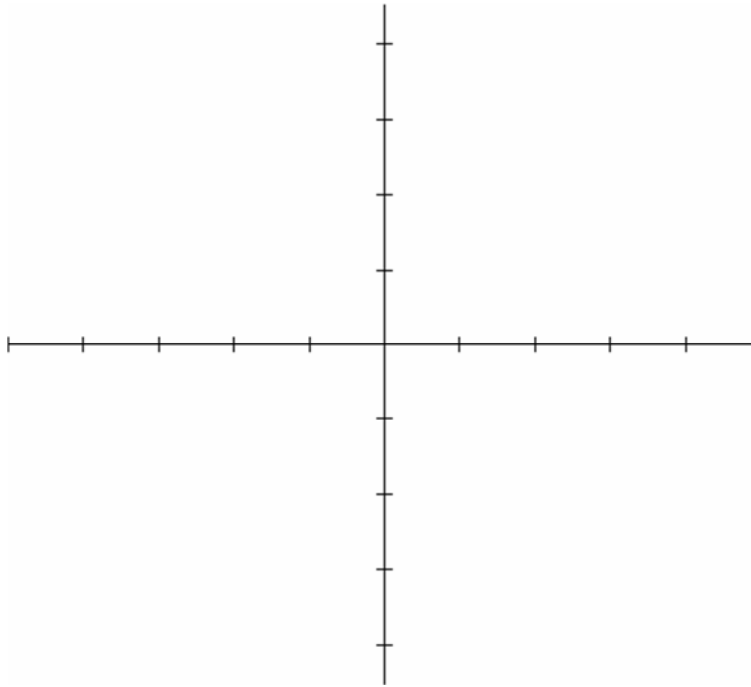$f(x) = \sqrt{x}$

$f(x) = x^3$

$f(x) = x^{1/3} = \sqrt[3]{x}$

$f(x) = \dfrac{1}{x}$

$f(x) = |x|$

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Example 7: Sketch the graph of $f(x) = \sqrt{x+2} + 1$

You should also be able to look at a function and state the transformations needed to convert a basic graph into the one that is requested.

Example 8: Describe how the graph of g is obtained from the graph of f.

a. $f(x) = \sqrt{x}$ $g(x) = \sqrt{x} - 3$

b. $f(x) = x^3$ $g(x) = -(x+2)^3 + 1$

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Another method to help sketch a graph of function is to determine whether it is even, or odd.

Even Function
A function, f, is even if $f(-x) = f(x)$ for all x in the domain of f. The graph of an even function is symmetric with respect to the y-axis

Odd Function
A function, f, is odd if $f(-x) = -f(x)$ for all x in the domain of f. The graph of an odd function is symmetric with respect to the origin.

Example 9: Determine if $f(x) = 5x^4 - 3x^2 + 2$, is odd, even or neither.

Example 10: Determine if $f(x) = x^3 - x$, is odd, even or neither.

Example 11: Determine if $f(x) = x^3 - x + 1$, is odd, even or neither.

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