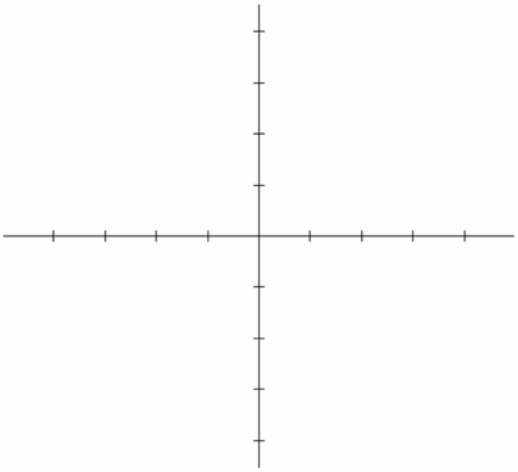
Math 1310
Section 3.5: Maximum and Minimum Values

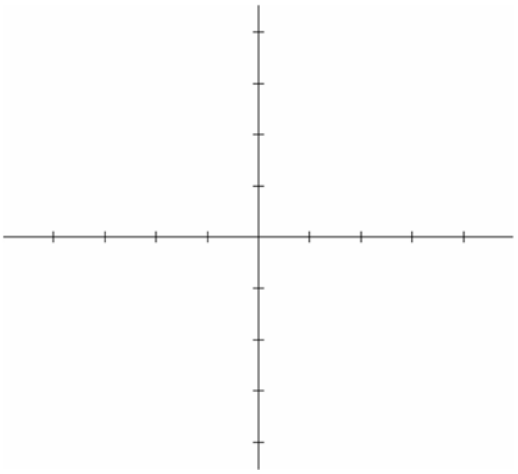
A quadratic equation is of the form $f(x) = ax^2 + bx + c$, where a, b, and $c$are real and $a \neq 0$

We have seen the graphs of **parabolas**.

If $a > 0$ then the parabola will open upwards.



If $a < 0$ then the parabola will open downwards.



Note: The larger $|a|$, the narrower the parabola

The **vertex** is the turning point of the parabola and is the **minimum point** on the graph when it opens upward and the **maximum point** on the graph when it opens downward. Every parabola has a maximum or minimum, but **NOT** both.

The **axis of symmetry** is a vertical line through the vertex that divides the graph in half.

The Standard form of a Quadratic Function

The quadratic function $f(x) = a(x-h)^2 + k$ is in **standard form**

The vertex is the point $(h,\ k)$ and the axis of symmetry is $x = h$

The domain is $(-\infty, \infty)$.

The range is $[k,\ \infty)$ if $a > 0$ or $(-\infty,\ k]$ if $a < 0$

Our first task will be to change a given quadratic function from the form $f(x) = ax^2 + bx + c$ to standard form. We'll complete the square to do this. Once the function is in standard form, we can sketch a graph using transformations and then read off the maximum or minimum value

Example 1: Write the following quadratic in standard form. Then find the vertex and the axis of symmetry.

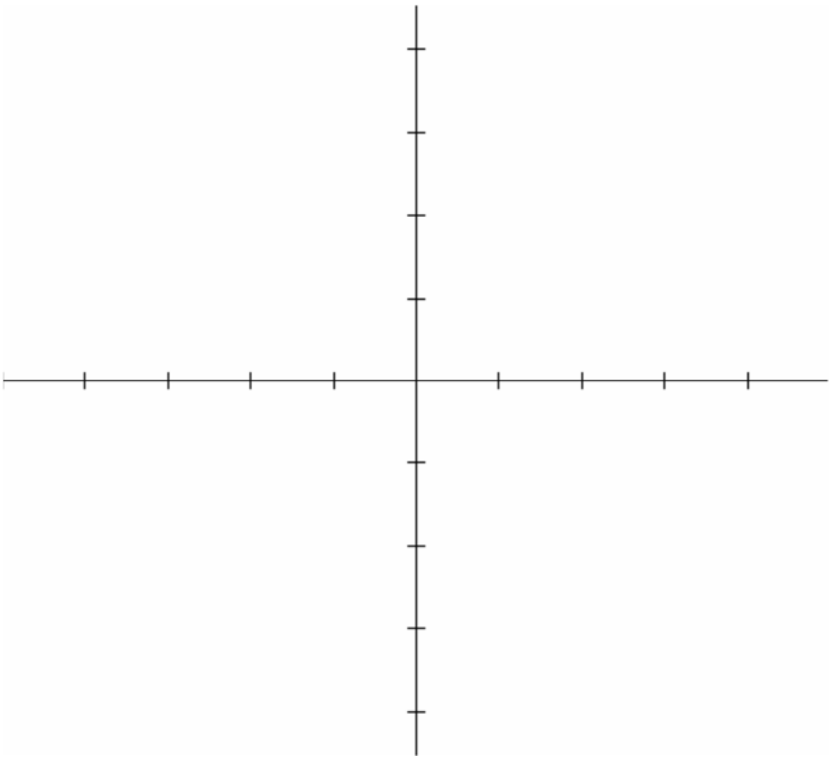
a. $f(x) = 3x^2 - 12x - 1$

b. $f(x) = -x^2 + 2x + 3$

Graphing Quadratic Functions with Equations in Standard Form

1. Determine whether the parabola opens upward or downward.
2. Determine the vertex.
3. Find any *x*-intercept by replacing *f*(*x*) with 0 and then solving for *x*.
4. Find the *y*-intercept by replacing *x* with 0.
5. Plot the intercept(s) and vertex, sketch the graph and draw the axis of symmetry.

Example 2: Sketch the graph of $f(x) = -x^2 + 2x + 3$



Shortcut:

For $f(x) = ax^2 + bx + c$, the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. So the axis of symmetry is $x = -\frac{b}{2a}$.

Example 3: Let $f(x) = 2x^2 + 4x + 7$. Determine, without graphing, whether the given quadratic function has a minimum or maximum value. Then find the coordinates of the minimum or maximum point

Example 4: Suppose $f\ (x) = \ 5x^2\ -\ 30x + 41$ Write the equation in standard form. State the coordinates of the vertex. Determine, without graphing, whether the given quadratic function has a minimum or maximum value. Then find the coordinates of the minimum or maximum point.

Finally, given the vertex or the x-intercepts of a quadratic function and one other point that lies on the graph of the quadratic function, you should be able to write the quadratic function.

Example 5: Find a quadratic function with vertex (2, 6) which passes through (-1, 4).

Example 6: Find a quadratic function with *x*-intercepts -4 and 3, with a y-intercept of 6.