

Math 1310**Section 3.6: Combining Functions**

Suppose we have two functions $f(x)$ and $g(x)$. The domain of $f(x)$ is the set A . The domain of $g(x)$ is the set B . We can combine these two functions together in five different ways:

Sum of Functions

Difference of Functions

Product of Functions

Quotient of Functions

Composition of Functions

We will discuss the first four

Sum of Functions: $(f + g)(x) = f(x) + g(x)$ with domain $A \cap B$

Difference of Functions: $(f - g)(x) = f(x) - g(x)$ with domain $A \cap B$

Product of Functions: $(fg)(x) = f(x)g(x)$ with domain $A \cap B$

Quotient of Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x - 4$. Find each of the following and state the domain:

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(fg)(x)$

d. $\left(\frac{f}{g}\right)(x)$

Example 2: Let $f(x) = x^2 - 3x - 1$ and $g(x) = -3x - 10$. Find

a. $(f + g)(1)$

b. $(gg)(-1)$

The Composition of Functions

The composition of the function f with g is denoted $f \circ g$ by and is defined by the

$$(f \circ g)(x) = f(g(x))$$

The domain of the composition $f \circ g$ is the set of all x such that

1. x is in the domain of g (the “inside” function)
2. $g(x)$ is in the domain of f (the “outside” function)

Example 3: Let $f(x) = x^2 + 1$ and $g(x) = -2x + 5$, find $(f \circ g)(x)$.

Example 4: Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{5}{x+4}$, find $(g \circ f)(x)$

Example 5: Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{3 - x}$, find $(f \circ g)(x)$.

Example 6: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

a. $(f \circ g)(2)$

b. $(g \circ f)(-1)$

c. $(g \circ g)(0)$