

## Math 1310

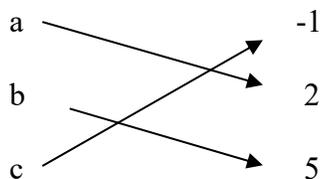
### Section 3.7: Inverse Functions

Let  $f$  be a function with domain  $A$ .  $f$  is said to be **one-to-one** if no two elements in  $A$  have the same image.

Example 1: Determine if the following function is one-to-one.

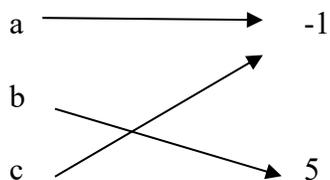
a.

Domain       $f$       Range



b.

Domain       $g$       Range



A one-to-one function has an inverse. The inverse function reverses whatever the first function did. These two statements mean exactly the same thing:

1.  $f$  is one-to-one (1-1)
2.  $f$  has an inverse function

The inverse of a function  $f$  is denoted by  $f^{-1}$ , read “ $f$ -inverse”.

Note:  $f^{-1}(x) \neq \frac{1}{f(x)}$  like  $x^{-3} = \frac{1}{x^3}$

#### Domain and Range

Suppose  $f$  is a one-to-one function with domain  $A$  and range  $B$ . The inverse function has domain  $B$  and range  $A$ .

**Example 1:** Suppose  $f$  and  $g$  are inverse functions. If  $f(3) = -1$  and  $f(-1) = 4$ , then find  $g(-1)$ .

### Property of Inverse Functions

Let  $f$  and  $g$  be two functions such that  $(f \circ g)(x) = x$  for every  $x$  in the domain of  $g$  and  $(g \circ f)(x) = x$  for every  $x$  in the domain of  $f$  then  **$f$  and  $g$  are inverses of each other.**

**Example 2:** Show that the following functions are inverses of each other.

$$f(x) = 3x + 7 \text{ and } g(x) = \frac{x}{3} - \frac{7}{3}$$

**Example 3:** Determine whether the following pair of functions are inverses of each other.

$$f(x) = 2x - 1 \text{ and } g(x) = \frac{x}{2} + 1$$

**How to find the equation of the inverse function of a one-to-one function:**

1. Replace  $f(x)$  by  $y$ .
2. Exchange  $x$  and  $y$ .
3. Solve for  $y$ .
4. Replace  $y$  by  $f^{-1}(x)$
5. Verify.

**Example 4:** Write the equation of the inverse function for  $f(x) = 3x - 3$

**Example 5:** Write the equation of the inverse for  $f(x) = \frac{6}{4-x}$

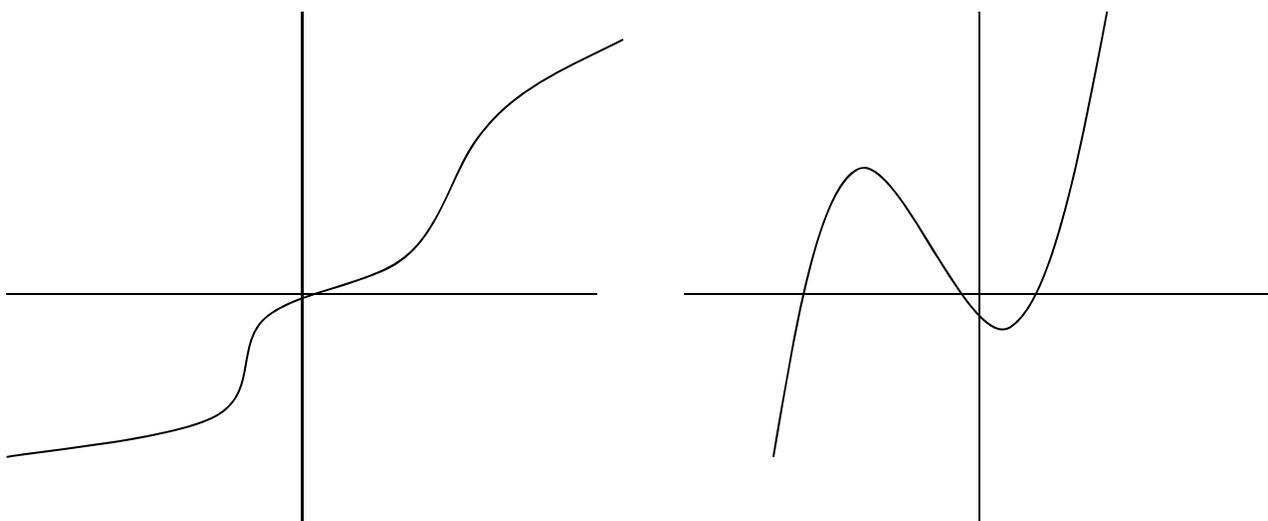
**Example 6:** Write the equation of the inverse for  $f(x) = (2x + 1)^3 + 1$

**Example 7:** Write the equation of the inverse for  $f(x) = \frac{2x+3}{x-4}$

It is easiest to determine if a function is one-to-one by looking at its graph. We can use the Horizontal Line Test to determine if a function is one-to-one.

**Horizontal Line Test:** A function is one-to-one if no horizontal line intersects its graph in more than one point.

**Example 8:** Is the following graph the graph of a function that has an inverse function?



### Graphing the Inverse Function

Given that  $f$  is 1-1, the graph of  $f^{-1}$  is a reflection of the graph of  $f$  about the line  $y = x$

Remember:

1. The inverse function reverses whatever the first function did.
2. The Domain of  $f$  becomes the Range of  $f^{-1}$  and the Range of  $f$  becomes the Domain of  $f^{-1}$ .

**Example 9:** Below is the graph of  $f$ . Use this graph to draw the graph of its inverse function.

