Math 1310
Section 5.1/5.2: Exponential Functions and the Number *e*

Functions whose equations contain a variable in the exponent are called **exponential functions.**

Real-life situations that can be described using exponential functions:

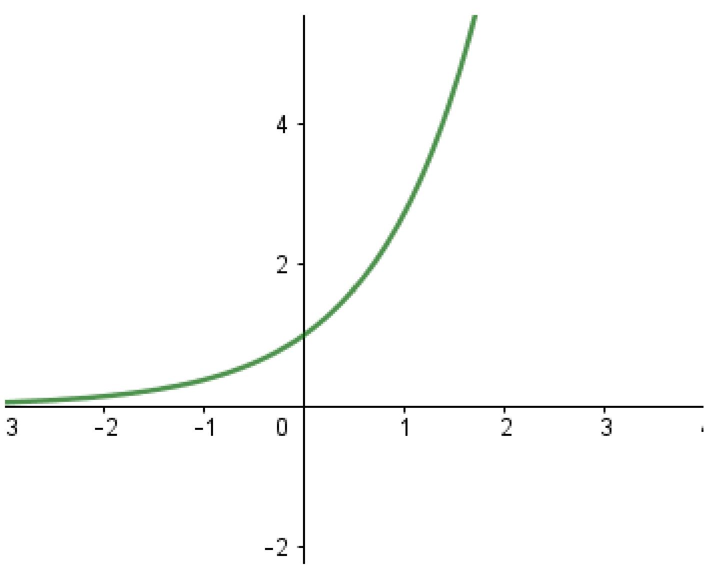
1. population growth
2. growth of an epidemic
3. radioactive decay
4. other changes that involve rapid increase or decrease

The function $\boldsymbol{f(x) = a^x}$ is the exponential function with base $a > 0$ and $a \neq 1$.

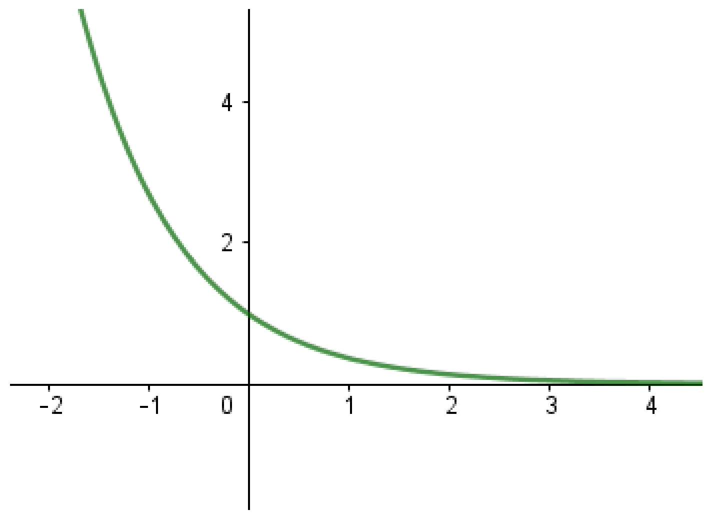
We'll be interested in graphing exponential functions. What you already know about graphing functions using transformations will apply.

We'll look at two cases of the exponential function, $a > 1$ and $0 < a < 1$.

For $a > 1$:
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key point: $(0, 1)$
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to -\infty$
The graph of $f(x) = a^x$ with $a > 1$ has this shape (larger a results in a steeper graph):



For 0< a < 1:
Domain: (- ∞,∞)
Range: (0, ∞)
Key point: (0, 1)
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to \infty$
The graph of $f(x) = a^x$ with $0 < a < 1$ has this shape (smaller a results in a steeper graph):



Example 1: Sketch the graph of $f(x) = 2^{x+1} - 3$ by transforming the graph of $f(x) = 2^x$. State the domain, range, asymptote and translation of the key point.

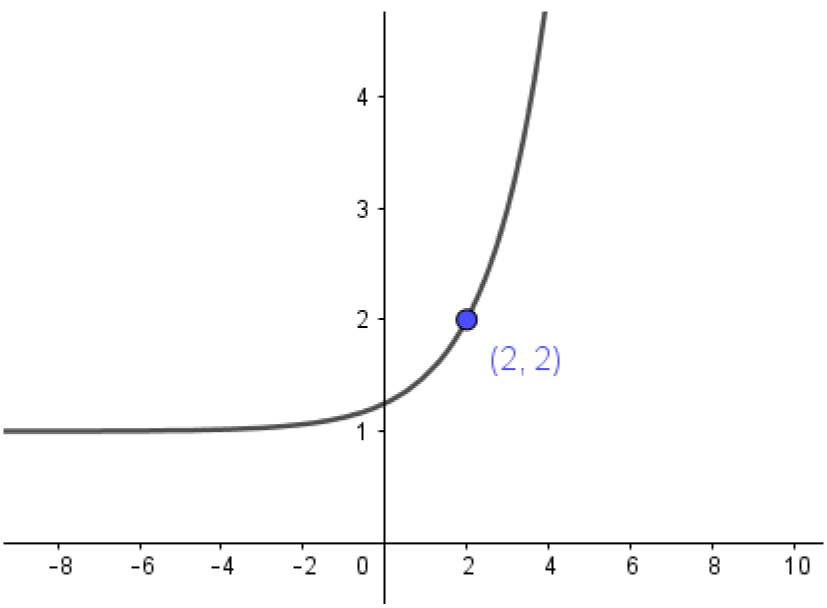
Example 2: Sketch the graph of $f(x) = -2^{x+1} + 2$ by transforming the graph of $f(x) = 2^x$. State the domain, range, asymptote and translation of the key point.

Example 3 Sketch the graph of $f(x) = (2/3)^x - 2$ by transforming the graph of $f(x) = (2/3)^x$. State the domain, range, asymptote and translation of the key point.

Sometimes you'll just be given a couple of points that lie on the graph of the function. You can use the same method to find the equation.

Example 4: Suppose $f(x)$ is an exponential function which passes through (0, 1) and (3, 125). Find $f(x)$.

Example 5: Given the graph, determine the function associated with it.



Section 5.2: The number *e*

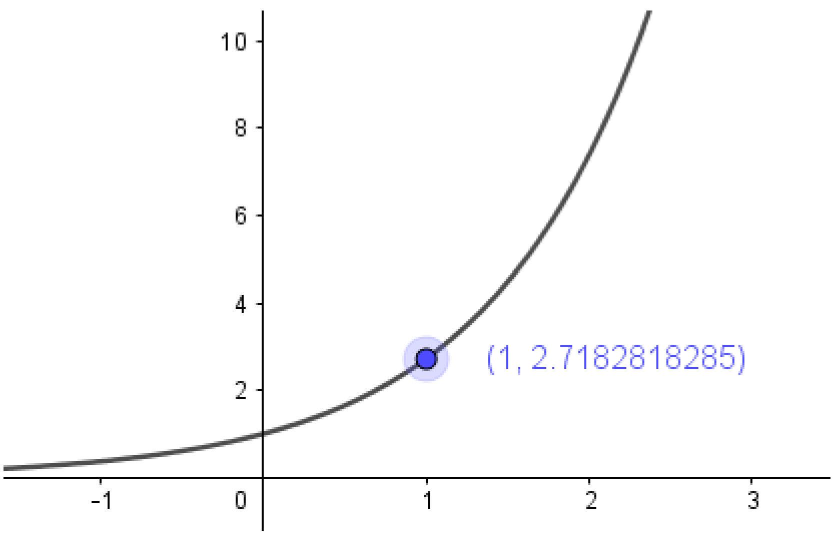
Definition: e is the "limiting value" of $\left(1+\frac{1}{x}\right)^x$ as x grows to infinity.

$e \approx 2.718281282459$. It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal.

Since $e > 1$, e can be the base of an exponential function. So everything we learned in Section 5.1 about graphing exponential functions will apply to graphing the function $f(x) = e^x$

The graph of $f(x) = e^x$ will have the following features:
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key point: (0, 1)
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to -\infty$

Here is the graph of $f(x) = e^x$:



Example 1: Sketch the graph of the function of $f(x) = -e^{x+2} + 2$ using transformations. State the domain, range, asymptote and translation of the key point.