Math 1310
Section 5.3: Logarithmic Functions

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base a is called the **logarithmic function with base *a*.**

For $x > 0$ and $a > 0$ and a not equal to 1, $y = \log_a x$ is equivalent $a^y = x$

The function $f(x) = \log_a x$ is the **logarithmic function with base *a***

The **common logarithm** is the logarithm with base 10. We denote this as $\log_{10} x = \log x$
The **natural logarithm** is the logarithm with base e. We denote this as $\log_e x = \ln x$

You will find both of these logarithms on a scientific calculator.

Note: We do not typically write either $\log_{10} x$ or $\log_e x$.

Example 1: Write each equation in its equivalent exponential form.

a. $3 = \log_6 x$

b. $2 = \log_a 64$

c. $\log_3 27 = 3$

d. $\log 100000 = 5$

e. $\ln \frac{1}{e^2} = -2$

Example 2: Write each equation in its equivalent logarithmic form.

a. $4^3 = 64$

b. $2^6 = 64$

c. $e^x = 25$

d. $10^x = 1000$

Example 3: Evaluate, if possible.

$\log_6 36$

$\log_2 \frac{1}{8}$

$\log_5 125$

$\log 100$

$\log_4 2$

$\log 0.001$

$\log_3\left(\sqrt[3]{81}\right)$

$\log_5 \sqrt[4]{125}$

Inverse Property of Logarithms

For $a > 0$ and $a \neq 1$

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

Example 4: Evaluate.

a. $\log_{14} 14^3$

b. $5^{\log_5 34}$

c. $e^{\ln 32}$

d. $\log_{47} 47^{\pi}$

Recall that for $x > 0$ (and $a > 0$ and a not equal to 1), we have $f(x) = \log_a x$. So the domain of $f(x) = \log_a x$ consist of all x for which $x > 0$.

Example 5: Find the domain.

a. $f(x) = \log_2(x - 2)$

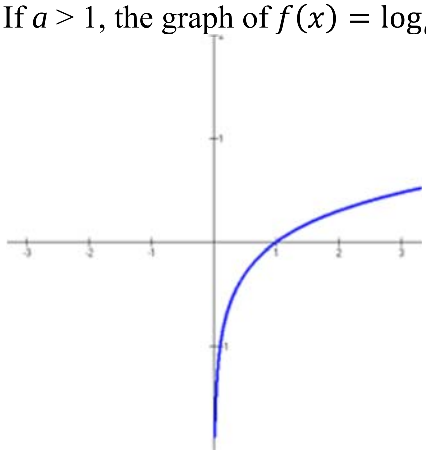
b. $f(x) = \ln(7 - 2x)$

c. $f(x) = \log(x^2 + 1)$

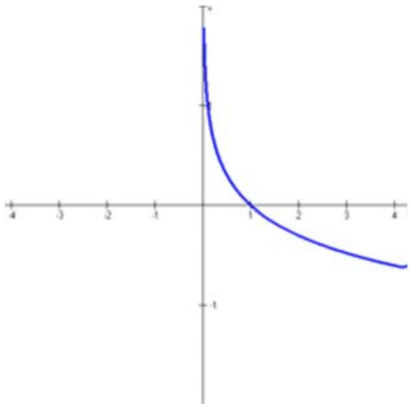
Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_a x$

1. The x-intercept is (1, 0) and there is no y-intercept.
2. The y-axis is a vertical asymptote.
3. The domain is all positive real numbers.
4. The range is all real numbers.

If $a > 1$, the graph of $f(x) = \log_a x$ looks like:



If 0< a < 1, the graph of $f(x) = \log_a x$ looks like:



Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

Example 6: Sketch the graph of $f(x) = \log_4(x+2)$. State the domain, range, asymptote and key point.

Example 7: Sketch the graph of $f(x) = -\ln(x-1) + 1$. State the domain, range, asymptote and key point.